Cat bonds & Artificial Neural Networks | An example of reinsurance products’ pricing using machine learning methods

By Mikaël Benizri | Actuary | Ensae ParisTech

Supported by Ziad Fares | R&D

Global Research & Analytics

1 This work was supported by the Global Research & Analytics Dept. of Chappuis Halder & Co.
# Table of contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>3</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>4</td>
</tr>
<tr>
<td>1. THE CAT BONDS MARKET</td>
<td>5</td>
</tr>
<tr>
<td>1.1. XL Contracts &amp; Cat Bonds</td>
<td>5</td>
</tr>
<tr>
<td>1.1.1. XL Contracts</td>
<td>5</td>
</tr>
<tr>
<td>1.1.2. Cat bond</td>
<td>6</td>
</tr>
<tr>
<td>1.1.3. Trigger</td>
<td>8</td>
</tr>
<tr>
<td>1.1.4. Differences between XL contracts and Cat bonds</td>
<td>8</td>
</tr>
<tr>
<td>1.1.5. Modelling Agency</td>
<td>9</td>
</tr>
<tr>
<td>1.2. Cat bonds markets: actors and their motivations</td>
<td>10</td>
</tr>
<tr>
<td>1.3. Market size &amp; perspective of evolution</td>
<td>10</td>
</tr>
<tr>
<td>2. WIND CATASTROPHES MODELLING IN SOUTHEASTERN UNITED STATES</td>
<td>14</td>
</tr>
<tr>
<td>2.1. Overview of geographic context and historical data</td>
<td>14</td>
</tr>
<tr>
<td>2.2. Modelling and prediction of annual costs</td>
<td>15</td>
</tr>
<tr>
<td>2.2.1. Data analysis: principal component analysis (PCA)</td>
<td>15</td>
</tr>
<tr>
<td>2.2.2. Modelling with the Artificial Neural Network method</td>
<td>16</td>
</tr>
<tr>
<td>2.2.3. Application: Regression by neural network and predictions</td>
<td>18</td>
</tr>
<tr>
<td>2.2.4. Sensitivity of the model to explanatory variables</td>
<td>21</td>
</tr>
<tr>
<td>2.3. Modelling and prediction of the annual number of catastrophes and of the annual costs associated</td>
<td>22</td>
</tr>
<tr>
<td>2.3.1. Data analysis: principal component analysis (PCA)</td>
<td>23</td>
</tr>
<tr>
<td>2.3.2. Application: Classification by neural network and predictions</td>
<td>25</td>
</tr>
<tr>
<td>2.3.3. Sensitivity of the model to explanatory variables</td>
<td>30</td>
</tr>
<tr>
<td>3. APPLICATION TO HERITAGE INSURANCE COMPANY</td>
<td>32</td>
</tr>
<tr>
<td>3.1. Heritage Insurance Reinsurance program</td>
<td>32</td>
</tr>
<tr>
<td>3.2. XL contract: pricing methods</td>
<td>34</td>
</tr>
<tr>
<td>3.3. Application: Heritage Insurance reinsurance program</td>
<td>35</td>
</tr>
<tr>
<td>3.4. Pricing of the last cat bond issued by Heritage Insurance</td>
<td>37</td>
</tr>
<tr>
<td>3.4.1. Pricing methods for cat bonds</td>
<td>37</td>
</tr>
<tr>
<td>3.4.2. Application to the last Cat bond issued by Heritage Insurance</td>
<td>38</td>
</tr>
<tr>
<td>CONCLUSION</td>
<td>40</td>
</tr>
<tr>
<td>TABLE OF FIGURES</td>
<td>42</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>43</td>
</tr>
<tr>
<td>CONTACTS</td>
<td>44</td>
</tr>
</tbody>
</table>
Cat bonds & Artificial Neural Networks | An example of reinsurance products’ pricing using machine learning methods

Abstract

Over the last fifty years, numbers and costs of natural disasters have not ceased to multiply. Given this phenomenon, insurers and reinsurers struggle to cover the associated losses. Consequently, they turned to financial markets in order to obtain new hedging capabilities, by using various types of products, such as excess of loss contracts (named XL) and cat bonds.

This paper presents a mathematic model allowing to predict the number and the cost of incoming catastrophes. Data used include wind catastrophes affecting the southeast area of the United States and whose damages are worth more than a billion dollars. This model helps to price insurance risk transfer products, such as XL contracts or cat bonds. First a regression relying on neural network methodology is implemented in order to predict the global annual cost of future catastrophes. Then, based on the same methodology, a classification is done in order to allocate these costs to the various catastrophes.

Our models are used to price several contracts included in the reinsurance program of “Heritage Insurance,” so our results help estimate the share of premiums received by the reinsurer. Two calculations methods are applied: the exposure curve and the “burning cost” method.

Our models are also validated on cat bond products, but this time using a financial method. This method allows to estimate the share of the Expected Excess Return (EER) depending of the Probability of First Loss (PFL) and the Conditional Expected Loss (CEL).

Keywords: Natural disaster, XL contract, cat bond, sponsor, Special Purpose Vehicle (SPV), insurance market, financials market, swap, zero-β asset, Sharpe Ratio, ACP, non-linear regression, classification, neural network, multi-layers perceptron, training data, test data, linear regression, time series, sensitivity

JEL Classification: G1, G22, C45, C52, C53
Introduction

In 2013, natural disasters caused more than 25,000 deaths and economic damages in the world worth more than 140 billion dollars (Sigma Re, 2014). Additionally, the fifth report from the intergovernmental expert group on climate change claims that a raise of the intensity and frequency of natural disasters should be expected. Consequently, costs associated to natural disasters may follow that trend. Currently, only thirty percent of the damages caused by natural disasters are covered by insurance contracts. In this context, the insurance world is looking for new hedging capabilities.

To meet the specific needs of liquidity, the insurance market looked towards the financial markets. In 1994, the first cat bonds were introduced on the “Chicago Board of Trade.” Indeed, financial markets have an almost unlimited capacity, which means they can cover any costs generated by a catastrophe. This difference of hedging capacity between these two markets is directly correlated to their size. For instance comparing the size of the reinsurance market with the monetary and sovereign debt market highlights the fact that financial markets are the solution for the insurance market. According to the French Federation of the Insurance Companies, in 2011 the global volume of premiums for reinsurance represented 220 billion dollars whereas the monetary and sovereign debt market was estimated at 8 trillion dollars.

However, natural disasters are hardly predictable as well as their associated costs. Consequently, investors in financial markets are unwilling to invest in the insurance area since they are not used to this practice. They are not ready to position themselves on products that they don’t understand and whose valuation is not transparent. In order to fight against this opacity and to take into account the degree of uncertainty linked to natural disasters, investors must be able to price insurance risk transfer products, such as cat bonds.

That’s why our effort resulted in the development of a statistical model allowing to estimate and predict the number and costs of future catastrophes; our whole models are constructed on public data. These describe wind catastrophes from the southeast areas of the USA. Forecasts obtained through our models are focused on this area and type of catastrophe in order to price insurance risk transfer products (XL contracts or cat bonds). The model is highly inspired by the article “La survenue des catastrophes naturelles: classification des variables explicatives par les réseaux de neurones” written by Rim Jenli, Nouri Chtourou, Rochdi Feki and Damien Bazin in 2012. In this article, the authors try to predict the number of natural disasters by leveraging neural networks.

First, this article presents the market of cat bonds, its operation and potential evolution. After describing our modelling methodologies, results are used to price the reinsurance program of the company “Heritage Property and Casualty” as well as their last cat bond.
1. The Cat Bonds Market

The core principle behind insurance lies in the pooling of risks. Insurers provide insurance contracts and are carriers of the risks. Some risks are well known by insurers, such as in the automotive sector, while others are less mastered, for instance natural disasters risks. In a perspective of stabilizing their results, insurers attempt to keep in their portfolios claims with a low severity and a high frequency since they are easier to model. In the opposite way, they try to avoid claims with high severity and low frequency since they are harder to model. Indeed, if a catastrophe of a high severity occurs in the year, costs related to loss hedging will highly increase and this will impact results of the insurer. In order to limit this type of risk, insurers rely on reinsurers. Reinsurance is insurance for insurance companies. The insurer provides a portion of premium received to the reinsurer. These premiums cover situations whose occurrence is rare and cost high. The reinsurer carries the risk instead of the insurer thanks to excess of loss contracts (XL contracts). The figure below illustrates this situation.

![Figure 1 Example of insurance risk transfer](image)

**1.1. XL Contracts & Cat Bonds**

**1.1.1. XL Contracts**

An excess of loss contract, also called XL contract, is a risk transfer agreement between an insurer and a reinsurer. The insurer pays a portion of its premiums to the reinsurer in order to help manage a part of their potential losses. Following the occurrence of a claim, if the losses reach a given amount, called the priority or retention, the reinsurer assumes the losses exceeding this amount up to a limit called the scope. The figure below illustrates three various cases for a contract 30 XL 20 (in million dollars) following the occurrence of a claim.

![Figure 2 Example of a 30XL20 contract](image)

The first case corresponds to the occurrence of a claim whose cost (60 million dollars) exceeds the scope of the contract. The reinsurer carries the capacity of the contract (30 million dollars) instead of the insurer. The insurer bears 30 million dollars which correspond to the sum of the priority (20 million dollars) and the amount exceeding the scope (10 million dollars).
The second case corresponds to a claim whose cost (30 million dollars) exceeds the priority of the contract, but without reaching the scope. The insurer bears the retention (20 million dollars) and the reinsurer assumes 10 million dollars which is the difference between the cost of the claim (30 million dollars) and the retention (20 million dollars).

The last case corresponds to a claim which never reaches the priority of the contract. The insurer bears the totality of the costs and the reinsurer is not involved.

Similarly to insurers, reinsurers try to transfer a part of the risks they carry. That is why they also use XL contracts from other reinsurers. To compensate for their lack of funding they also turn to financial markets and use products such as cat bonds.

1.1.2. Cat bond

Cat bonds (for catastrophes bonds) are bonds called “High Yield” which are mainly issued by insurance or reinsurance companies. They work the same way as classic bonds with a close difference: the payment of coupons and the nominal repayment is conditioned upon the occurrence of a certain type of catastrophe in a certain area and determined duration. The repayment of the nominal can be partial or total. This type of obligation allows insurance and reinsurance companies to transfer risks associated to these exceptional events to third parties. This effectively reduces their own risk.

The basic operation of a cat bond is the following: The sponsor, i.e. the company that wishes to transfer its risks, directly or indirectly issues a bond debt specifying the terms of repayment. In the indirect case, the issuance is made via a special purpose vehicle (SPV).

The repayment term varies depending of the type of catastrophe, the area or the hedging duration (generally between 1 and 5 years). The conditional repayment may concern either coupons, interest payments or both simultaneously. On signing the contract, investors deposit the funds required for purchase in a secured account. This amount is then blocked. The sponsor can access it only if the triggering event (i.e. the natural disaster) occurs. In order to pay for this deposit, the sponsor pays interest to the investors generated by the investment of the bond value. If during the hedging period, the catastrophe on which the bond is indexed occurs, the funds are used to repay the damage generated by the catastrophe. It is also possible and sometimes more interesting from a fiscal point of view to go through an intermediate structure such as a special purpose vehicle (SPV) or a special purpose reinsurer (SPR). The SPR provides to the insurer a standard reinsurance contract funded by the issuance of the cat bond. The funds and the premiums paid by the insurer are invested in very low-risk assets. Investors are only exposed to the risks that the insurer wishes to cover.
Figure 3 functioning of a cat bond

The figure above shows the functioning of an issuance via a SPV. The sponsor provides a portion of its premium to the SPV, which issues bonds to the investors. The SPV uses the money from the investors and the premiums from the sponsor to generate revenues. In parallel, it contracts a swap giving it access to a fixed income in order to pay the investors (coupons). If the catastrophe occurs, the SPV uses the investor’s money to pay the sponsor.

Figure 4 Cat bond – Step by Step

The figure above summarizes the different steps in the life of a cat bond:

- **Step 1** | The sponsor gives a share of its premiums to get a right of hedging. The SPV invests this money in very low-risk and liquid assets. Additionally, it contracts a swap in order to generate fixed income.
- **Step 2** | As long as no catastrophe occurs, the SPV pays a coupon to the investors on which the cat bond is indexed.
- **Step 3** | If a catastrophe occurs whose cost covers the value of the bond, investors are no longer paid and the sponsor receives the amount of the hedging from the SPV.
1.1.3. Trigger

The concept of catastrophe occurrence is defined by the cat bond trigger. There are different types:

- **Indemnity trigger** | These triggers correspond to the exceeding of a threshold of loss from the sponsor. They limit the risk by ensuring the amount received to cover the losses incurred deviates from the actual amount of losses.

- **Trigger related to sectorial index** | They are based on indices of catastrophe occurring in some specific sectors. For instance, insurance-indices exist to represent the losses incurred by the insurance industry.

- **Trigger related to modeled indices** | They are the weighting of sectorial indices.

- **Parametric trigger** | They are based on the level of some parameters such as the wind speed or parametric indices which are a function of these parameters.

- **Hybrid trigger** | They are the combination of the previous triggers.

These triggers have various advantages, according to the relevant participants. The sponsor will rather use indemnity triggers indexed on its real losses. Investors will rather use transparent triggers in order to clearly estimate their amount of loss. Besides, they need to know as soon as possible the exact amount of their losses. Indeed, they work in a short-term approach environment. In an indemnity context, the process of loss estimation can last for a long period.

1.1.4. Differences between XL contracts and Cat bonds

Different elements distinguish traditional reinsurance from securitization of insurance risks. Traditional reinsurance allows to cover almost all types of risks. Triggers used in traditional reinsurance are indemnity triggers: payments of claims correspond to the actual losses of the sponsor. It is also possible to perform reconstruction of guarantees. The price and the capacity covered remains uncertain over several years. The reinsurer represents a counterparty, legal and operational risk for his client. Indeed, the counterparty risk depends on the ability of the reinsurer to meet its commitments. On the contrary, securitization of insurance risks is primarily devoted to natural disasters. Various types of triggers are used and may generate a basis risk. The price and the capacity covered are fixed over many years. Legal, operational and counterparty risks are negligible.

The table below summarizes the main differences between traditional reinsurance and securitization products such as cat bonds:

<table>
<thead>
<tr>
<th>Reinsurance (XL)</th>
<th>Securitisation of risks (Cat Bond)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Available for almost any type of risk</td>
<td>• Risks covered: mainly natural events</td>
</tr>
<tr>
<td>• Payment of the claims according to the real cost assumed by the sponsor.</td>
<td>• Payment of the claims according to the type of trigger:</td>
</tr>
<tr>
<td>- Indemnity trigger</td>
<td>- Indemnity</td>
</tr>
<tr>
<td></td>
<td>- Insurance index</td>
</tr>
<tr>
<td></td>
<td>- Sectorial index</td>
</tr>
<tr>
<td></td>
<td>- Modeled index</td>
</tr>
<tr>
<td></td>
<td>- Parametric index</td>
</tr>
<tr>
<td>• Reconstitutions of guarantees possible</td>
<td>• Reconstitutions complicated</td>
</tr>
<tr>
<td>• Price and capacity uncertain on several years</td>
<td>• Price and capacity fixed on several years</td>
</tr>
<tr>
<td>• Counterparty risk depending on the rating of the reinsurer</td>
<td>• Counterparty risk reduced or even removed</td>
</tr>
<tr>
<td>• Legal and operational risks</td>
<td>• Legal and operational risks reduced</td>
</tr>
</tbody>
</table>

Figure 5 Differences between reinsurance and securitization of insurance risks
1.1.5. Modelling Agency

In order to price cat bonds, investors and reinsurers rely on the works of the three biggest modelling agencies, namely RMS, EQE and AIR. Risk Management Solution (RMS), founded in 1988 at Stanford University, helps insurers, financial markets, companies and administrations to estimate and take over risks from natural disasters around the world. Established in 1994, EQECAT is a risk modelling company delivering products and services to insurers, reinsurers and financial markets. Finally, AIR Worldwide Corporation, founded in 1987, models the risk of natural disasters and terrorism in more than 90 countries.

Although they use the same stochastic methods and approaches, and base this on the same data, these three agencies often get different results. Given this discrepancy, investors and reinsurers will prefer one of these three agencies depending on the type of the natural disaster and the area considered. As it shown in Figure 6 between 1990 and 2007, most disasters involved floods, storms or epidemics.

![Figure 6: Type of natural disasters in the world between 1990 and 2007](http://www.notre-planete.info/terre/risques_naturels/catastrophes_naturelles.php)

Figure 6 below shows the distribution in volume of cat bonds issued in 2009, depending on the agency who modeled it. It is observed that AIR agency is the market leader.

![Figure 7: Distribution of cat bonds modeled by agencies](source: DUBREUIL E. [2010] "La titrisation des grands risques : Evolution ou Revolution ?")
1.2. Cat bonds markets: actors and their motivations

In order to understand the functioning of the cat bonds market, as well as its possible evolution, it is important to analyze the motivations of the different players.

**Insurers and reinsurers:** As explained previously, insurers and reinsurers are looking for alternative solutions to traditional reinsurance. Most of them seek mainly for an additional hedging capacity in order to meet their commitments, specifically in extreme situations.

Financial markets work differently than the insurance market. On the latter, which is smaller, the reputation effect to its customers is much more important. That is why, insurers and reinsurers need to be sure that they can meet their commitments. Consequently, on financial markets they are ready to offer coupons at a higher rate in order to give themselves the hedging capacity that will enable them to answer their client’s needs.

Additionally, with the arrival of Solvency II, another advantage of the cat bonds is that they reduce the capital needed as reserve requirements compared to traditional reinsurance. Indeed, reimbursements from traditional reinsurance contracts depend on the reinsurer capacity to pay indemnities, whereas the funds from the issued cat bonds is held at contract signature and can only be used to reimburse sponsor losses. The securitization transaction implemented when issuing a cat bond helps to limit, or even eliminate, the counterparty risk.

**Investors:** From the perspective of investors, cat bonds are attractive for many reasons. Their main advantage, in portfolio managements, come from the fact that they are not correlated, in theory, with the performance of financial markets (namely “zero-beta” assets). Indeed, the fall of a stock index won’t cause any natural disasters but the inverse relationship is uncertain. For instance, the occurrence of an earthquake in a major metropolitan area will directly impact the economic and financial activities of a country, and consequently financial markets. In a portfolio diversification goal, this type of product is particularly sought after. Moreover, these bonds payout at higher rates than government bonds, and this is especially interesting in the current financial environment where rates are low. Cat bonds allow to increase significantly the ratio between profitability above the risk-free rate and the risk of financial portfolios (namely Sharpe ratio).

1.3. Market size & perspective of evolution

Over the last fifty years, the number of natural disasters has increased drastically. A natural disaster is an extreme climatic event. The classification of a climatic event such as a natural disaster and its coverage varies by country. In France, for instance, a natural disaster is defined by an administrative entity, with recognition by the Ministry of the Interior, and allows for the systematic compensation to victims.

The main reasons behind the increase in the number of natural disasters around the world are global warming, pollution, greenhouse gases and climate cycles leading to devastating storms. However, these findings must be nuanced. Indeed, the increase in the number of natural disasters is also due to the new ways of measuring them. Thus, many disasters occurring in the middle of the ocean have been recorded thanks to new technologies like satellites. Areas are also now more populated so if a disaster occurs in these areas it will be recorded.
Overall, the cost of damage generated by natural disasters has been multiplied by 5 over the last forty years. Indeed, major disasters have occurred in developed countries where the advancements in infrastructure and equipment has greatly increased the cost of damages.

The frequency and the cost of natural disasters became so high for insurance and reinsurance companies that they needed to find new hedging capabilities. Catastrophes are more difficult to cover and the demand of insurance is increasing. The volume of insurance premiums tripled between 1990 and 2013, from USD 84bn to USD 240bn. Insurers and reinsurers have been focusing on financial markets in order to transfer a part of their risk, such as leveraging cat bonds, but this explosive growth in premiums does not appear to be slowing down.
Considering the market players, main issuers of cat bonds are reinsurers; but direct insurers represent a significant share, as well as manufacturers and the government. For instance, in France, main issuers are Scor, AXA, Groupama and EDF.

On the side of investors, before the 2007 crisis, the distribution was approximately:
- 30% “cat funds” (investment funds specialized in natural disasters)
- 30% institutional investors
- 20% reinsurers
- The rest was divided between “Hedge funds” and mutual funds.

Since the crisis, the distribution has evolved:
- It is only 18% institutional investors and 13% reinsurers
- “Cat funds” represents now 40%
- “Hedged funds” are now approximately at 30% whereas they were almost absent before the crisis

Cat bond markets now show record volumes. The market of insurance securitization reached USD 23bn in 2014. This market is mainly comprised of cat bonds, but alternative solutions of risk transfer also exist. Moreover, transactions done on this market are mostly private, so it is complicated to estimate its size. According to the estimation of Swiss Re, Goldman Sachs and Aon, the whole market represents approximately USD 50bn and is expected to double within the next 5 years.

While focusing on the part of damage covered during the occurrence of natural disasters, it is observed that only one third of the cost are supported by insurance companies. Consequently, it is still necessary to find more hedging capabilities, leading many to believe that the size of the cat bonds market will keep growing.

The access to products such as cat bonds is not the same for every country. On average, only 3% of the damages due to a natural disaster are covered in developing countries. In order to facilitate the access to the cat bonds market, the World Bank implemented a platform for issuance of cat bonds. It was created in 2009 in partnership with Mexico, one of the most experienced country in risk management related to natural disasters. This country issued several cat bonds to cover its risk of earthquakes in 2006. The program, named “MultiCat,” allows governments and public entities to insure themselves on affordable terms.

Although the number of players in the cat bonds market keeps growing, many obstacles hinder its development. First, rating agencies, like Standard & Poor’s, don’t take into account all the advantages of cat bonds, such as their high return or the diversification they bring in a portfolio as a zero-beta asset. The financial model approach used by rating agencies to assess the capital needed for insurance companies doesn’t consider these advantages. The rating of cat bonds is capped at “BB,” which is considered as speculative securities by Standard & Poor’s. Besides the solvency level of issuers is under rated since it is the status (SPV) that is rated and not the entity.
Another restraint to the expansion of this market lies in the complexity of traded products. Indices or models used for cat bonds are hard to understand. In theory, these indices must be transparent, simple, viable and available and published by independent structure. In practice, they are very technical and difficult for investors to understand. Lastly, difficulties arise from the lack of standardization of cat bonds. Indeed, many are exchanged via the over-the-counter market, which creates opacity in information.

In this context, the modelling of natural disasters and the pricing of insurance risk transfer products is a major issue for all market players. Indeed, investors are not keen to buy cat bonds, whose characteristics remain uncertain and unknown. That’s why this paper proposes a way to model and evaluate these products in order to help investors.
2. Wind catastrophes modelling in southeastern United States

In our model, we focus in wind catastrophes that happened in southeastern United States, because a majority of recent cat bonds relate to this geographic area and this type of catastrophe. The model is based on these data. It could have been set on another area or a different kind of catastrophe. The main objective is to present a modelling tool that can be changed and used by investors in different contexts, and for different geographic areas. We model frequency and severity of catastrophes and then we make predictions about them for the next three years. The results are used in order to price reinsurance contracts such as XL contracts and cat bonds. The selected data originated from the National Climatic Data Center, include only wind catastrophe events (storm, hurricane, tornado) which lead to more than 1 Billion dollars of economic damages.

2.1. Overview of geographic context and historical data

Southeastern United States is an environment conducive to wind catastrophes for meteorological reasons. It is a subtropical humid area. Annual temperature is above 15°C. Summers are hot and humid because of the ascents of tropical air masses from Gulf of Mexico (average temperatures in July is above 22°C). Winters are mild thanks to the Gulf-Stream. In late summer, those regions are affected by storms and hurricanes, including cyclones from the Antilles. This geographic area has thus been strongly affected by meteorological catastrophes for several years.

The type of catastrophes and their intensities are varied. There are low intensity events like winter storms that cause lower costs. Other events have higher intensity and generate much higher costs. For instance, the category 4 hurricane Andrew that occurred in 1992 cost 46 billion dollars. Some storms have been particularly violent such as the “Storm of Century” in 1993 also called “93 Super Storm.”

Data are composed of events that occurred during the period from 1980 to 2014. Those events seem randomly spread over time, except for the 2004-2005 period. During this period, Florida has been affected by 7 high intensity hurricanes. It has begun with the category 4 hurricane called Charley that caused 21 billion dollars of damages. Then, category 2 hurricane Francis occurred, it is infamous for the degradation of the NASA’s Kennedy Space Center. Only three weeks after, category 3 hurricane Jeanne caused 9 billion dollars of damages and killed 4 people. Then, after the category 3 hurricane Denis, the tragic hurricane Katrina occurred. It has been the most expensive and deadliest one. 1245 people died and the costs reached 151 billion dollars. Winds sustained 280km/hour. Florida has then been affected by two other hurricanes, Rita which was as big as Germany and Wilma that caused the closure of the airport in Miami for several days.

Figure 11 Study area
Modelling and prediction of annual costs

The model used in order to predict the annual costs of damages generated by wind catastrophes in the southeastern United States is a non-linear regression based on an artificial neural network. The data used in the model are economic and demographic ones such as the size of the population (number of persons that may own goods to cover) and the house prices (can be assimilated with part of the amount of covered goods). They help describe the evolution of the costs. Finally, the explanatory variables are:

- Number of citizens residing in the state of Florida
- Median wage of citizens in the state of Florida
- Real estate index
- Leading index (6-month forecast for growth rate)
- Federal income tax
- Federal income tax per citizen living in the state of Florida
- Percentage change of the Federal income tax per citizen living in the state of Florida

These variables are calculated on an annual basis. The dependent variable that we want to explain is the annual costs of wind catastrophes on the southeastern United States.

2.2.1. Data analysis: principal component analysis (PCA)

In order to describe our explanatory variables, we have performed a principal component analysis. This method consists of a reduction of dimensions in the data base. It allows to describe the existing links between the quantitative variables that are used in the model.

These variables contribute to the construction of the axis of the principal component analysis. Indeed, they correspond to linear combination of these different variables. Once the axes are obtained, the cumulative costs of each year are then projected on the different axes.

In a first step, the correlation circle illustrates the variables that have contributed to the construction of the axis. The proximity of the arrowheads with the circle indicates that the variables are well represented.

Figure 12 Correlation circle

The first axis is made up of the variables “Federal income tax” (25.1%), “Number of citizens residing in the state of Florida” (25%), “Federal income tax per citizen living in the state of Florida” (25.2%) and “Real estate index” (20.8%). The second axis is composed of “Percentage change of the Federal income tax per citizen living in the state of Florida” (14.8%).
tax per citizen living in the state of Florida” (54.5%) and “Leading index” (43%). Thus, the first axis is made up of variables that describe the Florida economic situation at a point of time. The second one takes into account the evolution of the economic index.

The next graphic illustrates how the annual costs are placed on these axes. Each point represents the cumulative costs in a year. Points 1 to 7 corresponds to the period 1991-1999 and the points 8 to 14 to the period 2003 to 2013. The horizontal axis splits the annuals costs by decades: the points to the left correspond to the nineties, the ones to the right represent the years after 2000. The first axis distinguished the number of citizen as well: the more a point is on the right, the higher the number of citizens. The level of costs is indicated by the vertical axis: the higher a point is on this axis, the higher the costs.

**Figure 13 Annual costs according axis 1 and 2**

The values of points’ cosine-squared allow to confirm the quality of representation of the points on the two first dimensions. Indeed, for the 14 points, the sum of the $\cos^2$ is close to 1 (except for point 8, the year 2003). It means that they are well represented by the two axes.

### 2.2.2. Modelling with the Artificial Neural Network method

A neural network corresponds to a combination of more or less complex elementary objects, called neurons, allowing to handle non-linear problems. A neuron is a model characterized by an internal state, input signals and a potential of activation. The latter is the weighted sum of the different input signals. The input signals correspond to the explanatory variables for the first layer, and then to their transformation due to the first layer for the second one, and so on. The result provided by each neuron is then used by all the neurons of the next layer. There are different types of neural networks, characterized by their architecture, the number of hidden layers, the number of neurons by layer, their transfer function and by the type of result desired (continuous or categorical). The application of the transfer function is what obtains the next layer of neurons.

We focus on the multi-layer perceptron that corresponds to a succession of hidden layers (generally one or two). Within these layers, neurons are not linked, but they are linked to every neuron of the previous layer and to those of the next one. The following drawing represents such a structure.
For instance, we describe a given path of the explanatory variable $x_1$. This variable goes from the input layer to the first neuron of the first hidden layer (1). Within this neuron, the combination of $x_1$ with all the other explanatory variables lead to the creation of a new variable, through the activation function of the neuron. The result is then sent to each neuron of the next hidden layer (2). Within each neuron, the combination of this result and the results coming from the other neurons of the previous layer creates a new value through the activation function. Finally, the different values obtained come together in the last neuron, the output layer (3). The combination of these values through the transfer function allows to obtain the estimated value of the dependent variable we want to predict. The variable $x_1$ follows the same type of path by getting through every neuron of the first hidden layer.

As in the case of regression with one hidden layer composed of $q$ neurons, the dependent variable to explain is as follow:
The coefficients are estimated so that the quadratic error is minimized.

\[
\min_{\alpha, \beta} \sum_{i=1}^{n} (y_i - \phi(X; \alpha, \beta))^2
\]

The minimum is determined by the assessment of the gradient by back-propagation. It is used to calculate the derivatives of \( EQ(\alpha, \beta) \)

\[
\frac{\partial EQ_i}{\partial \beta_k} = -2(y_i - \phi(x_i))(\beta'V_i)V_{k,i} = \delta_i V_{k,i}
\]
\[
\frac{\partial EQ_i}{\partial \alpha_{k,j}} = -2(y_i - \phi(x_i))(\beta'V_i)\beta_k f'(\alpha'_{k,j})x_{i,p} = s_{k,i}x_{i,p}
\]

With \( V_{k,i} = f(\alpha_{k,0} + \alpha'_{k} x_i) \) et \( V_i = \{V_{1,i}, ..., V_{q,i}\} \)

The terms of this equation are evaluated in two stages. A first estimation of the \( y_i \) is made. The obtained values are compared with the real ones. It gives us the \( \delta_i \). The latter are then back-propagated and allow to calculate the \( s_{k,i} \). The evaluation of the gradients is then obtained. A gradient descent algorithm is used to find the optimal coefficients.

\[
\beta_{k}^{j+1} = \beta_{k}^{j} - \theta \sum_{i=1}^{n} \frac{\partial EQ_i}{\partial \beta_k}
\]
\[
\alpha_{k,p}^{j+1} = \alpha_{k,p}^{j} - \theta \sum_{i=1}^{n} \frac{\partial EQ_i}{\partial \alpha_{k,p}}
\]
\[
\theta_{j+1} = \epsilon \theta_j, \text{ where } \epsilon \in [0, 1[\]

The learning rate \( \theta \) decreases according to the \( \epsilon \) parameter in order to progressively refine the results when the algorithm gets closer to a solution.

2.2.3. Application: Regression by neural network and predictions

The neural network method is used to define our model. In order to use the model as a predictor, we use projections of the explanatory variables on the three next years, provided by the “Office of Economic and Demographic Research.” For the variables “Real Estate index” and “Leading index” for which the predictions are not available, we have projected them by time series methods using ARIMA models.
The latter help deal with non-stationary series after determining their level of integration (number of time we have to differentiate the series to make it stationary).

The data finally obtained by combining the predictions provided by the “Office of Economic and Demographic Research” and by the time series constitute the prediction base.

In order to determine the most relevant model, which means to determine the number of hidden layers, number of neurons by layer, the complexity parameters (learning rate for instance), the database (except predictions) is divided into two sub-bases: a learning base on which the model is calibrated and a test base on which the model is tested.

The methodology used works as follows: the model is tested with different combinations of parameters to determine the optimal parameters and then used to make the predictions of future annual costs. The next graphic illustrates the different steps set up.

---

**Figure 16 Calibration methodology**

For each combination of parameters, steps 1 to 3 are made in order to determine the final model that is used as a predictor in steps 5 and 6:

- **Step 1** | The model is calibrated on the learning base. This is the way the coefficients of the model are estimated, conditionally to the parameters set.

- **Step 2** | Once the model is calibrated, the explanatory variables of the test base that were not used for the calibration are used as input. We then obtain the estimation of dependent variables for the test base.

- **Step 3** | The predicted values obtained on the test base are compared with the real values. The mean square error (MSE) is then calculated:

\[
EQM = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2}
\]

- **Step 4** | For each combination of parameters, steps 1 to 3 are made. The combination of parameters finally chosen is the one that minimizes the mean square error.

- **Step 5** | Once the parameters are chosen, the coefficients are estimated on the entire database (combination of the learning sub-base and the test sub-base). This takes into account all the information available before making the predictions.

- **Step 6** | The model is then used to estimate the future values of the dependent variable. The input variables are the prediction of the explanatory variables. They represent the prediction
The model used does not converge. But it converges on the mean value when 100 replications are made (In our case, 100 replications are sufficient for the mean to converge). The obtained results correspond to the mean of all simulations of each year.

The most accurate results are obtained with a regression using a neural network composed of one hidden layer with 27 neurons, having sigmoid functions as activation functions and a linear one as a transfer function.

For a given year $i$,

$$
\hat{Y}_i = \phi \left( X_i ; \hat{\alpha} ; \hat{\beta} \right)
= \hat{\beta}_0 + \hat{\beta} V
$$

Where $V = \begin{pmatrix}
1 & \frac{1}{1+e^{(a_1x_1)}} & \cdots \\
\vdots & \vdots & \ddots \\
1 & \frac{1}{1+e^{(a_{27}x_1)}} & 
\end{pmatrix}$

The learning base is composed of data over the period 1991-2009, the test base on 2010-2013 and the prediction base on 2014-2018. We do not have the data for the explanatory variables for 2014 (not yet available) but we already know the annual cost of the wind catastrophes for this year, equal to 4 Billion dollars. The fact that the prediction base has a known value allows to verify how well the model predicts the first value. The next graphic illustrates the results obtained on the test base of the chosen model.

![Figure 17 Results from the Test sub-base](image)

On the learning base, the model almost perfectly reproduces the data, it presents a risk of over fitting. Nevertheless, it remains the model most powerful for the test base. It is then retained.

The annual costs predicted for the year from 2014 to 2018 are:

<table>
<thead>
<tr>
<th>Year</th>
<th>Forecasted costs (USD bn)</th>
<th>Observed costs (USD bn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>3.74</td>
<td>4</td>
</tr>
<tr>
<td>2015</td>
<td>13.6</td>
<td>-</td>
</tr>
<tr>
<td>2016</td>
<td>4.38</td>
<td>-</td>
</tr>
<tr>
<td>2017</td>
<td>12.2</td>
<td>-</td>
</tr>
<tr>
<td>2018</td>
<td>9.01</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 1 Forecasted costs**

We notice that the first value is really close to the observed cost (3.74 and 4 Billion dollars).
2.2.4. Sensitivity of the model to explanatory variables

We are now conducting the analysis of the sensitivity of the chosen model to explanatory variables. It helps identify the input variables that have a strong impact on the output of the model, but also the ones that have less influence on these outputs. The sensitivity is measured by comparing the predictions of the costs on the test base with and without each variable.

When the variables “Federal income tax”, “Number of citizens residing in the state of Florida”, “Federal income tax per citizen living in the state of Florida” and “Median wage of citizen of the state of Florida” are successively taken out of the model, the effect on the prediction of the costs is similar. The costs predicted with one of the variables out of the base have the same trends as the full model. However, the estimation is less accurate and the estimated values are less close to the real cost than when all explanatory variables are used as input. Therefore, these four variables participate to the predicting capacity of the model.

![Figure 18 Sensitivity – Case 1](image)

By taking out one by one the variables “Real Estate index” and “Percentage change of the Federal income tax per citizen living in the state of Florida” of the database, the impact on the prediction of the costs by the model is strong. Indeed, the order of the costs is preserved, but the order of magnitude is not well predicted. Hence, these two variables help to distinguish the level of costs associated to the catastrophes. By taking out these variables, the predicting power diminishes.
By taking out the variable “Leading index,” the order of the predicted costs is not maintained. Indeed, for the second year, without this variable, the predicted cost is higher than the one of the first year. It is less high in the case where the database contains all the variables, and the real case. This variable contributes to predicting the level of costs associated with the catastrophes.

2.3. Modelling and prediction of the annual number of catastrophes and of the annual costs associated

The objective of this section is to allocate the annual costs provided by the previous model. To do so, we set up a classification model using neural networks which aim to detect catastrophes. The explanatory variables used are monthly meteorological measures:

- Average temperature
- Maximum temperature
- Minimum temperature index
- Precipitation index
- Heating Degree Days
- Cooling Degree Days
- Palmer Drought Severity Index
- Palmer Hydrological Drought Index
- Palmer Z-index
- Modified Palmer Drought Severity Index
- Land Temperature Anomalies (south hemisphere)
- Ocean Temperature Anomalies (south hemisphere)
- Land Temperature Anomalies (north hemisphere)
- Ocean Temperature Anomalies (north hemisphere)
2.3.1. Data analysis: principal component analysis (PCA)

In order to describe the explanatory variables of this second database, we have performed a principal component analysis once again. For this one, 4 axes have to be utilized. Thus the PCA is made on the first four dimensions. The correlation circle below represents the variables on the first two axes. The variable “Precipitations index” is located between the two axes. The two first dimensions do not allow to take into account the effect of this variable. The latter highly contributes to axis 5, which is not kept.

Regarding the contribution of each axis, the first one is mainly composed of the variables “Cooling Degree Days” (21.9%), “Average temperature” (21.8%) and “Maximum temperature” (21.1%). For the second dimension, the main variables that constitute the axis are “Palmer Drought Severity Index” (19.8%), “Palmer Hydrological Drought Index” (18%), “Modified Palmer Drought Severity Index” (19.7%) and “Palmer Z-index” (12.3%).

The first axis takes into account the different temperatures (minimum, maximum, average). The second one indicates the level of drought in the area. These two axes well distinguish the points by the level of costs of the catastrophe. Indeed, the catastrophes to the far right are the ones that generated the largest costs (orange and red points).

The correlation circle for axes 3 and 4 indicates that one variable seems to be essential for axis 4: “damages.” The quality of representation is very high because the arrowhead of the variable is on the circle.
The third dimension is explained by the variables “Ocean Temperature Anomalies (north hemisphere)” (20.2%), “Palmer Drought Severity Index” (13.04%), “Land Temperature Anomalies (south hemisphere)” (20.8%), “Ocean Temperature Anomalies (south hemisphere)” (19.7%). The fourth dimension is almost only comprised of the variable “damages” (99.3%).

On the following graphic, representing the PCA for axes 3 and 4, the points distinctly separate the catastrophes by their cost with the x-axis. The less important catastrophes in terms of cost are at the bottom (grey and black points) and the ones with a high cost are at the top.

The values of points’ cosine-squared signifies the quality of representation of the points on the four dimensions. Indeed, for 30 points among the 33 points, the sum of the $\cos^2$ is close to 1. This means that they are well represented by the two axes.
2.3.2. **Application: Classification by neural network and predictions**

We set up a nested model, using the results of the previous one. In order to avoid accumulating several errors due to the forecasts of the explanatory variables using time series. Instead of predicting these variables, we made a delayed in time model. We try to explain the occurrence of catastrophes, and their layer of intensity, by the different measures taken 36 months prior.

The advantage of this method is that there is no need to make predictions of the explanatory variables of the prevision base of this classification model.

Let’s explain the technique used. The traditional method consists of using the explanatory variables $X$ at the date $t_0$ in order to explain and estimate the dependent variable $Y$ at the same date $t_0$. For instance, we have used this method for our first model.

![Figure 25 Dependent variable according to the explanatory variables – Conventional case](image1)

Now, for the second model, a time lag has been set up. The explanatory variables of the date $t_{-1}$ are used to explain and estimate the dependent variable $Y$ at the date $t_0$. The lag between these two dates is 36 months.

![Figure 26 Dependent variable according to the explanatory variables – Time delayed case](image2)

This helps avoid errors by doing predictions of the future explanatory variables. Indeed, we use known explanatory variables and we explain the dependent variable $Y$ for the 36 next months in the future.

![Figure 27 Forecast base in the case of a time delayed model](image3)

With the same objective as the first model, and with an aim of validation, the database is divided into two sub-bases as well, a learning one and a test one. We keep the model with the highest share of correct
predictions on the test sub-base. We consider a value well predicted if the catastrophe is seen in the corresponding year and if its associated cost is in the same order of magnitude.

The chosen model is a classification neural network with one hidden layer composed of 35 neurons. The neurons have a sigmoid function as activation function and as transfer function for the output layer as well.

For a given month \(i\), \(\forall j \in \{1, \ldots, 15\}\) : 

\[
\hat{Y}_{ij} = \phi(X_{i-36} ; \alpha_j ; \beta_j) = \frac{1}{1 + e^{-\beta_j V}}
\]

Where

\[
V = \begin{pmatrix}
\frac{1}{1+e^{-\alpha_1 X_{i-36}}} \\
\vdots \\
\frac{1}{1+e^{-\alpha_{15} X_{i-36}}}
\end{pmatrix}
\]

We have defined 15 levels of intensity for the natural catastrophes that constitute our database. This intensity is taken for the costs generated by the catastrophes. The catastrophes obtained with the classification are ordered by level of intensity in the output matrix. Moreover, the catastrophes are placed according to the month, one line corresponds to a month. Each column corresponds to a neuron from the output layer associated with a layer of intensity. The output layer is thus composed of 15 neurons.

The structure of the network is in the following form:

![Figure 28 Artificial Neural Network – Classification model](image)

In order to understand how the time placement of the catastrophes and their associated cost have been determined, the output matrix \(Y\) has been dealt as follow.

The model does not converge, thus we make 100 successive simulations that give us 100 matrix \(Y\). This results in the average of the simulations and then helps obtain an estimator that converges.

The results come from a sigmoid function: its values are between 0 and 1. It is thus necessary to define a criterion to determine of which value a catastrophe is detected. The threshold has been defined so that the model predicts, on average, a good number of catastrophes on the test base. As a first step, the closest value to 1 by line is kept, representing each month. Then, it is transformed into a 1 if this value is bigger than the threshold. The chosen threshold is 0.3.
The drawing below presents an example of output for 6 months (Y from the test base corresponds in fact to 36 months). It gives us an example of predicted catastrophes by the model.

The first matrix corresponds to the first simulation. In this case, a catastrophe of intensity 3 is predicted at the second month, one of intensity 5 at the fifth month and one of intensity 7 at the third month. We reiterate this simulation and we sum the results with the previous ones. Thus, in our example below, the model predicts a catastrophe of intensity 3 at the second month and one of intensity 5 at the fifth month. This prediction has been made two times for the two iterations. It corresponds to situation A. In situation B, the model does not predict any catastrophe in the first simulation but it detects one on the second simulation. Finally, the model can predict a catastrophe during the first simulation (for instance at the third month, with intensity 7), but none during the second simulation: this is situation C. B and C complement one another. Between the first and the second simulation, the same catastrophe is detected, for a different layer of intensity but they remain close (level 7 for the first and 6 for the second). We sum the results of the 100 successive simulations. Moreover, for each simulation, we keep a record of the total number of catastrophes detected during the period, regardless of the intensity level. It allows us to calculate the average number of catastrophes on all simulations and then define the optimal threshold abovementioned.

**Figure 29 Methodology for detecting catastrophes and distributing costs – Step 1 to 3**

The catastrophes are ordered based on the number of times they have been detected during the 100 simulations. We keep a number of catastrophes equal to the average number of catastrophes detected during the 100 simulations.
In order to allocate the annual costs predicted by the first model to the different catastrophes detected by the second one, each catastrophe that the model considers detected (where there is a 1 in the matrix) is multiplied by the exponential of the number of its intensity layer. It improves the results on the learning sub-base and on the test sub-base.

Let $X$ be the average number of catastrophes detected by the model on the 100 iterations. In this example, $X=3$. Thus, only the three catastrophes that have been seen the most often are kept.

The results are then normalized on the catastrophes that are detected on the same year in order to obtain percentages of the annual cost that are going to be associated to each catastrophe. In this example, the only catastrophes detected are on the first six months of the year. Finally, the annual cost, predicted by the first model, is distributed according the percentages. In our example, the first model would have predicted 20 Billion dollars of cost for this particular year.

**Figure 30 Methodology for detecting catastrophes and distributing costs – Step 3 to 4**

**Figure 31 Methodology for detecting catastrophes and distributing costs – Step 4 to 5**
Using this methodology, the part of well predicted values on the learning sub-base is 95.4% and 83.3% on the test base.

On the learning base, 16 of 18 catastrophes on the period from 1980 to 2012 are detected. The order of magnitude of the costs are acceptable for most of them.

On the test base, composed of the years from 2012 to 2015, the model predict a good number of catastrophes. The location in time of the catastrophes is correct too because they all are in the right years. Only one catastrophe is seen at the exact month but there is the right number of catastrophes per year, and this is what we were looking for. The order of magnitude of the costs are good for all of the catastrophes, except for the first one. We have to notice that there is only one catastrophe detected during this year, so the model allocates all the annual cost predicted by the previous model to this catastrophe. The difference of cost comes from the first model, not from a bad allocation made by the second model. The result are satisfying.
Now, we use the data of the 36 last months of our database in order to make predictions for the three
next years, from July 2015 to June 2018. The model predicts 6 catastrophes that cost between 1.29 and 9.58 Billion dollars.

<table>
<thead>
<tr>
<th>Year</th>
<th>Forecasted catastrophes (USD bn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015 (1)</td>
<td>1.36</td>
</tr>
<tr>
<td>2015 (2)</td>
<td>9.58</td>
</tr>
<tr>
<td>2015 (3)</td>
<td>1.29</td>
</tr>
<tr>
<td>2015 (4)</td>
<td>1.29</td>
</tr>
<tr>
<td>2016</td>
<td>4.38</td>
</tr>
<tr>
<td>2018</td>
<td>9.01</td>
</tr>
</tbody>
</table>

**Figure 34 Forecasted catastrophes 2015-2018**

2.3.3. Sensitivity of the model to explanatory variables

We are now conducting the analysis of the sensitivity of the chosen model to explanatory variables, the
same way we did with the first model, by comparing the predictions on the test base with one explanatory
variable removed.

When the variable “Ocean Temperature Anomalies (north hemisphere)” is taken away from the base,
the performance of the model is better. The sensitivity analysis presented corresponds to the
sensitivities of the model readjusted after taking away this variable.

When the variables “Average temperature,” “Minimum temperature,” “Maximum temperature,”
“Precipitations index,” “Palmer Drought Severity Index,” “Palmer Hydrological Drought Index” and
“Palmer Z-index” are successively taken away of the model, it underestimates the number of
catastrophes predicted. These variables allow improve the model and increase the number of
catastrophes detected. These variables are sensitive to the occurrence of a catastrophe.
When the variables “Land Temperature Anomalies (south hemisphere)” and “Modified Palmer Drought Severity Index” are removed from the database, the number of catastrophes predicted is similar to the one with these variables. But by taking out the variable “Land Temperature Anomalies (north hemisphere),” the costs are not well distributed among the catastrophes and the predicted catastrophes are delayed in time. Indeed, the model estimates one catastrophe for the first year and three for the third year whereas the model with the full database predicts one catastrophe per year for the two first years and two for the last one.

Taking out the variable “Land Temperature Anomalies (south hemisphere)” leads to predict catastrophes in a similar way that with the full database. So, we cannot conclude on the impact of this variable.
3. Application to Heritage Insurance Company

Heritage Insurance is a public company based in Florida which is listed on the New York Stock Exchange. It is one of the ten biggest insurance companies in the state of Florida regarding wind events. In order to estimate the part of the costs generated by natural catastrophes that Heritage assumes, the amount of premiums the ten biggest companies have received has been gathered and normalized. These data stem from the investor presentation of Heritage insurance. This company represents around 10% of the catastrophe insurance market of Florida, as the following graphic illustrates:

![Figure 37 Heritage Insurance Company’s main competitors and market share](image)

According to the website “Argus de l’assurance”, the average part of damages generated by catastrophes that is insured worldwide was 30% in 2014 (34 of 113 Billions). Thereafter, the assumption of stability of this percentage is made for the three next years. Therefore, we make the assumption that Heritage insurance assumes 3% of the economic cost generated by the natural catastrophes it insures.

3.1. Heritage Insurance Reinsurance program

The reinsurance program of Heritage Insurance for the period 2015-2016 is composed of 11 contracts. These cover losses reaching 1.773 billion dollars. The program is divided into several layers. Within layer 2, Heritage Insurance has taken part at 75% in contract 4 and at 15% in contracts 5, 6 and 7.

<table>
<thead>
<tr>
<th>Retention (USD m)</th>
<th>Scope (USD m)</th>
<th>Part of the contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>contract 1</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>contract 2</td>
<td>35</td>
<td>119</td>
</tr>
<tr>
<td>contract 3</td>
<td>154</td>
<td>321</td>
</tr>
<tr>
<td>contract 4</td>
<td>360</td>
<td>200</td>
</tr>
<tr>
<td>contract 5</td>
<td>360</td>
<td>35</td>
</tr>
<tr>
<td>contract 6</td>
<td>360</td>
<td>650</td>
</tr>
<tr>
<td>contract 7</td>
<td>336</td>
<td>920</td>
</tr>
<tr>
<td>contract 8</td>
<td>475</td>
<td>150</td>
</tr>
<tr>
<td>contract 9</td>
<td>475</td>
<td>200</td>
</tr>
<tr>
<td>contract 10</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>contract 11</td>
<td>475</td>
<td>75</td>
</tr>
</tbody>
</table>

![Figure 38 Heritage Insurance Company’s reinsurance program](image)

There are no contracts which cover the claims with a very high cost. But it is offset by the fact that the less costly claims are covered by more contracts than necessary: the contracts are multiplied. Therefore, it “over-insures” the low costs claims in order to obtain liquidity to cover more costly claims.
For instance, as we can see on the following graphic, the occurrence of a catastrophe that generates 520 Million dollars of indemnity to be paid by Heritage Insurance triggers several contracts at the same time. It allows Heritage Insurance to receive a coverage of 836.25 Million dollars. All the contracts are triggered simultaneously, but only 5 of 11 are totally consumed.
The total capacity of coverage of Heritage Insurance is consumed when the amount of indemnity to be paid reaches 1.256 Billion dollars. Under our assumptions of market shares, it corresponds to a catastrophe that generate 58.5 Billion dollars of economic damages.

This capacity of coverage is 1.773 Billion dollars, including 15 Million of retention.

### 3.2. XL contract: pricing methods

In order to price the different types of risk transfer contracts included in the reinsurance program of Heritage Insurance, there are several methods.

- **Approximation using the Rate on Line** | The Rate on Line corresponds to the ratio between the premium of a contract and the covered capacity. It is calculated as the sum of the probabilities of reaching the priority of the contract and its coverage limit. It can also be valued as the inverse of the sum of return periods of events that generate damages reaching the priority and the coverage limit of the contract.

\[
\text{RoL} = \frac{\text{Premium}}{\text{coverage capacity}} = \frac{\mathbb{P}(\text{Loss} \geq \text{priority}) + \mathbb{P}(\text{Loss} \geq \text{coverage limit})}{2}
\]

\[
\text{RoL} = \frac{1}{\text{Return Period}[\text{priority}] + \text{Return Period}[\text{coverage limit}]}
\]

The premium is deduced from the RoL by multiplying the latter by the coverage capacity of the contract. The key point resides in estimating the probabilities of reaching the priority and the coverage limit of the contract. To do so, the traditional method in non-life insurance for low frequency and high severity catastrophes consists of using a Pareto distribution to model the costs and a negative binomial distribution for the frequency. Formerly, a Poisson distribution was used for the frequency. However, the increased number of catastrophes led to change the distribution because the equality \( \text{E}(\text{Frequency}) = \text{Var}(\text{Frequency}) \) is no longer consistent.

- **Exposure curve** | The exposure curve function, noted \( g \), is the ratio between the premium of the insurer and the base premium, \( g : [0,1] \rightarrow [0,1] \)

\[
g(c) = \text{Premium}_{+\infty XLc} = \mathbb{E}(X|X > c) = \frac{\mathbb{E}(\min(X,c))}{\mathbb{E}(X)}
\]

\[
bXLa = (+\infty XLa) - (+\infty XL(a + b))
\]

\[
\text{Premium}_{bXLa} = g(a) - g(a + b)
\]

With historical data, the function \( g \) is the value for a given threshold. Then, by making the priority (the threshold) varying, the exposure curve is obtained. This prices the contracts.

- **Burning cost** | The last method presented, the burning cost, consists of calculating the average part of claims’ costs in relation to the amount of premiums received each year, on a given time period. It is a non-parametric approach; no assumptions need to be made.

\[
C_j = \sum_{i=1}^{N_j} \min\left(b, \max(X_{ij} - a, 0)\right)
\]

\( C_j \) corresponds to the amount covered during the year \( j \) by the XL contract.

\[
\text{Rate}_{\text{burning cost}} = \frac{1}{n} \sum_{j=1}^{n} \frac{C_j}{\text{Premium}_j}
\]

34
This method is going to be used with our forecasts instead of our historic. The results will be compared to the one obtained with the exposure curve calibrated on historical data. The growth scenario used in order to estimate the future premiums received of Heritage Insurance is based on the historic growth of Heritage. Heritage has experienced significant growth over the last years. It is assumed that this expansion will continue during the next few years, but progressively in a less important way. The forecasts’ amounts of received premiums are:

<table>
<thead>
<tr>
<th>Year</th>
<th>Premiums (USD m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>43</td>
</tr>
<tr>
<td>2013</td>
<td>219</td>
</tr>
<tr>
<td>2014</td>
<td>436</td>
</tr>
<tr>
<td>2015</td>
<td>542</td>
</tr>
<tr>
<td>2016</td>
<td>573</td>
</tr>
<tr>
<td>2017</td>
<td>581</td>
</tr>
<tr>
<td>2018</td>
<td>583</td>
</tr>
</tbody>
</table>

**Figure 41 Premiums’ forecasts**

### 3.3. Application: Heritage Insurance reinsurance program

<table>
<thead>
<tr>
<th>Contract</th>
<th>Retention (USD m)</th>
<th>Coverage capacity (USD m)</th>
<th>Part of the contract</th>
<th>Premiums (Exposure curve)</th>
<th>Premiums (Burning cost based on forecast)</th>
</tr>
</thead>
<tbody>
<tr>
<td>contract 1</td>
<td>15</td>
<td>20</td>
<td>100%</td>
<td>1,40%</td>
<td>1,50%</td>
</tr>
<tr>
<td>contract 2</td>
<td>35</td>
<td>119</td>
<td>100%</td>
<td>4,90%</td>
<td>8,38%</td>
</tr>
<tr>
<td>contract 3</td>
<td>154</td>
<td>321</td>
<td>100%</td>
<td>7,03%</td>
<td>6,34%</td>
</tr>
<tr>
<td>contract 4</td>
<td>360</td>
<td>200</td>
<td>15%</td>
<td>0,41%</td>
<td>0,00%</td>
</tr>
<tr>
<td>contract 5</td>
<td>360</td>
<td>35</td>
<td>15%</td>
<td>0,08%</td>
<td>0,00%</td>
</tr>
<tr>
<td>contract 6</td>
<td>360</td>
<td>650</td>
<td>15%</td>
<td>1,22%</td>
<td>0,00%</td>
</tr>
<tr>
<td>contract 7</td>
<td>336</td>
<td>920</td>
<td>15%</td>
<td>8,60%</td>
<td>0,00%</td>
</tr>
<tr>
<td>contract 8</td>
<td>475</td>
<td>150</td>
<td>100%</td>
<td>1,79%</td>
<td>0,00%</td>
</tr>
<tr>
<td>contract 9</td>
<td>475</td>
<td>200</td>
<td>100%</td>
<td>2,39%</td>
<td>0,00%</td>
</tr>
<tr>
<td>contract 10</td>
<td>5</td>
<td>50</td>
<td>100%</td>
<td>3,19%</td>
<td>3,75%</td>
</tr>
<tr>
<td>contract 11</td>
<td>475</td>
<td>75</td>
<td>100%</td>
<td>0,00%</td>
<td>0,00%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1758</strong></td>
<td></td>
<td></td>
<td><strong>31,91%</strong></td>
<td><strong>19,97%</strong></td>
</tr>
<tr>
<td>+15 (retention)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>1773</strong></td>
</tr>
</tbody>
</table>

**Figure 42 Pricing of the reinsurance program of Heritage Insurance Company**

The calculation of the expected losses, based on our predictions, are point estimations and can be completed. Indeed, the results obtained with our models correspond to the averages of several simulations. These different simulations are used in order to determine the error distributions per year. From a reinsurer perspective that wishes to know if the rate of ceded premiums is satisfactory on average, the predictions are adjusted. The average positive error of the simulation, by year, is added to the predicted costs for each catastrophe. Only the unfavorable cases are taken into account. The average of the positive error is then added to the catastrophes. To do so, the probability that the cost associated to each year is underestimate is calculated, based on simulations, and the average error (in percentage) in this situation is deduced. Both parameters are multiplied and added to the cost of the predicted catastrophes.

\[
\text{Cost}_{\text{catastrophe}_{\text{year}_{t}}}^{\text{adjusted}} = \text{Cost}_{\text{catastrophe}_{\text{year}_{t}}} \times \{1 + E(\varepsilon|\varepsilon > 0) \times P(\varepsilon > 0)\}
\]

Where \(\varepsilon\) represents the error of predictions of the total cost of the year when the catastrophe happened. The following table compares the results obtained with the exposure curve based on historic and the ones with the burning cost method based on adjusted predictions.
In 2013, the premiums part ceded by Heritage Insurance in order to set up its reinsurance program was 32%. In 2014, it was 28.2%.
By using the exposure curve method, the estimated part of ceded premiums is 31.91% for the year 2015. With the burning cost method on the adjusted predictions, the obtained part is 33.02% whereas it was only 19.97% when it was calculated on point predictions. Although the distribution is quite different between the two methods, the estimations of ceded premiums ratios, 31.91% and 33.02% are in an acceptable order of magnitude (around 30%).

The predictions obtained with our neural network models allow us to approximate the part of ceded premiums necessary to set up the reinsurance program of Heritage Insurance. But results have to be moderated. Indeed, the part of ceded premiums by contract is not available: it is a private information. Even though the sum of ceded premiums is in an acceptable order of magnitude, it is impossible to verify if the distribution made by our model is relevant.

3.4. Pricing of the last cat bond issued by Heritage Insurance

After the pricing of XL contracts, the valuation of the last cat bond issued by Heritage Insurance is made. This second product helps submit our predictions to a new validation test. The aim is to verify that this modelling can be used to provide good indications to investors regarding the prices of cat bonds in which they are going to invest.

3.4.1. Pricing methods for cat bonds

Since one of the goals is to price cat bonds from an investor’s perspective, it is important to bring this product closer to financial markets products. Some financial products, especially the ones that cover credit risk, are similar to cat bonds. It is then possible to use pricing methods that are common on financial markets, in particular on the bond market, to price cat bonds.

As a first step, different methods of pricing proposed in scientific literature are presented, based on the note of Arthur Charpentier “Titrisation des risques catastrophes: les cat bonds de septembre 2002”. In the second step, one of these methods is applied to public data.

- **Actuarial method** | First, the actuarial method consists of assuming there is no free lunch. In this context, the price of the product is the sum of the adjusted future cash flows. The cash flows being random, the expected value has to be established, under a risk neutral probability. It consists of not using the historical probabilities of catastrophes occurrences but theoretical probabilities, in order to estimate the expected value. These probabilities make investors indifferent between putting their money in the bank with no risk or invest in cat bonds.

  \[ V_0 = E \left( \sum_{k=1}^{T} \frac{c(k)}{1 + r(k)} \right) \]

- **Financial method** | Another method applicable is the financial method. It corresponds to a method used on financial markets in order to value credit products such as CBO/CLO. It consists of determining the part of the expected return that is above the risk free rate (EER: Expected Excess Return) depending on the probability of occurrence (PFL: Probability of First Loss) and the expected loss given occurrence (CEL: Conditional Expected Loss). Morton Lane has illustrated this method (2000). Based on data from cat bonds issued during the period 1999-2000, he tried to build a linear model in order to link EER, PFL and CEL.

Based on the financial method logic, a linear model has been built, using recent available data from the website “artemis” which provides detailed descriptions of all cat bonds issued. The variables used in the linear regression are not the same that Morton Lane used but some are similar: attachment probability, exhaustion probability and expected loss (EL).

The final linear model that we obtained using data of the 30 last cat bonds issued is:

\[ EER = 0.0233 + 1.3807 \times EL \]
With adjusted-R2 equal to 95% and both coefficients significantly different from 0 with all usual threshold percentages (10%, 5%, 1%). The model has the advantage of being simple, only the expected loss of each layer of the cat bond has to be valued in order to price the cat bond. It also has a really high adjusted-R2, meaning that the variable EL well explains the levels of cat bonds’ coupons.

Figure 46 Cat bonds’ coupons – Estimations vs. observations

3.4.2. Application to the last Cat bond issued by Heritage Insurance

The cat bond studied here has been issued by Heritage Property and Casualty Insurance Company in April 2015, and took effect in June 2015 for a 3 years’ duration. It covers storms in the USA up to 277.5 million dollars. Initially, it only covered Florida. The trigger is an indemnity type, indexed on economic losses. The modelling agencies AIR Worldwide has been called for the valuation of covered disasters. The cat bond is split in three layers:

- **Layer A** | It is considered as the least risky. It activates at 410 million dollars of losses and covers up to 610 million dollars of losses.
- **Layer B** | It activates at 336 million dollars of losses and has a capacity of 650 million dollars. Its scope is of 986 million dollars.
- **Layer C** | It is considered as the riskiest. It activates at 336 million dollar of losses and covers up to 536 million dollars of losses. So it has a capacity of 200 million dollars.

Layer A is the least risky because its threshold of activation is the highest. It is less likely to be used than layer B or C. Layer B activates at the same threshold as layer C but has a higher capacity so it is less likely to be fully consumed. Layer B is considered as less risky than layer C.

To do the comparison, the study focuses on initial coupons, which means cat bonds that only cover storms in Florida. The final coupons are of the same order of magnitude as the initial. Forecasts obtained thanks to our neural network models are used in order to deduce the expected losses on each layer. These losses are compared to expected losses forecasted by AIR Worldwide, the agency who modeled this cat bond.

Once the expected losses are estimated, the linear model gives the following prices:

<table>
<thead>
<tr>
<th>Layer</th>
<th>Coupon</th>
<th>AIR</th>
<th>Neural Networks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer A</td>
<td>4.75%</td>
<td>3.92%</td>
<td>2.71%</td>
</tr>
<tr>
<td>Layer B</td>
<td>6.00%</td>
<td>5.60%</td>
<td>2.83%</td>
</tr>
<tr>
<td>Layer C</td>
<td>9.00%</td>
<td>9.21%</td>
<td>4.17%</td>
</tr>
</tbody>
</table>

Figure 47 Cat bond pricing using various methods
Using the expected losses estimated by AIR, an idea of the coupon’s order of magnitude is given. Considering our forecasts, expected losses are lower than AIR which results in obtaining coupons half priced.

As with the modelling of XL contracts, calculations are regenerated using forecasts adjusted with the mean positive error. Thanks to these new costs, expected losses are estimated and the following coupons are obtained.

<table>
<thead>
<tr>
<th>Neural Networks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer A</td>
</tr>
<tr>
<td>Layer B</td>
</tr>
<tr>
<td>Layer C</td>
</tr>
</tbody>
</table>

**Figure 48 Cat bond pricing taking into account the positive error**

Increasing the cost of disasters activates all layers, the coupon level difference is made on the capacities which reverse the order of layer A and B. Indeed, triggering probabilities of each layer is very close, the difference comes from the completion probability of each layer. Consequently, layer B appears as the least risky. Besides, the coupons obtained for layer A and C is higher than the real coupons.

<table>
<thead>
<tr>
<th>Low limit</th>
<th>High limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer A</td>
<td>2.71%</td>
</tr>
<tr>
<td>Layer B</td>
<td>2.83%</td>
</tr>
<tr>
<td>Layer C</td>
<td>4.17%</td>
</tr>
</tbody>
</table>

**Figure 49 Range of coupons**

If we assume that values of obtained coupons help define the range of the real coupons, then coupons of layer A are in the middle of its range, coupons of layer B are just above its upper bound, and the ones of layer C are nearly equal to its upper bound. From an investor’s perspective, based on these intervals, layer A doesn't look interesting but layer B and C are attractive: they deliver coupons corresponding to unfavorable scenarios or even worse for layer B.

**Figure 50 Real coupons and their estimated range**
Conclusion

Over the last years, the frequency and cost of natural disasters increased for many reasons, such as global warming or the use of new technologies to record catastrophes. Given this growth, insurers and reinsurers face difficulties. Indeed, they don’t have enough capacity to cover the cost of the damages incurred. Consequently, they are looking for new ways of funding and looked to the financial markets. The needs of hedging capacities transferred from the insurance market to financial markets via various products such as cat bonds. Then investors purchased these products and became a source of funding for insurers and reinsurers.

Recently, investors have gotten more and more interested in cat bonds. However, far away from the world of modelling natural disasters, they struggle to understand the full functionality of these products. So their pricing is a major issue. Currently, investors use information from rating agencies, but they tend to underestimate the rating of cat bonds. For instance, if agencies rated the issuer instead of the entity the rating will increase, and demand for these products will grow.

In order to price these particular products, we built a model with the goal of helping investors gain understanding and knowledge. In particular, it gives an order of magnitude of the coupons associated to cat bonds.

The model forecasts the frequency and the cost of future disasters. It prices insurance risk transfer products such as cat bonds or XL contracts. In order to analyze and validate the results obtained, they are compared to the reinsurance program of the company “Heritage Insurance.”

The model focuses on a specific area and type of disasters. But it can be adapted to any kind of situations. For instance, if an investor wishes to price a cat bond indexed on another type of disaster or geographic area, he can still use the methodology behind the model. In our paper, the model has been built on public data, to apply the model to another type of disaster it is mandatory to collect the correct data. Also, if an investor wishes to price a cat bond with another trigger, like a parametric trigger, he won’t use the model to predict the cost of the disasters but rather the value of the trigger. However, results are hardly generalizable since the model has been built on select data. Moreover, our model is based on the neural network methodology but it’s not a restriction since any other approach or machine learning methodology could be used to calibrate this model.

Finally, it is possible to compare our forecasted costs with the real costs observed. According to Munich Re, costs in 2015 generated by natural disasters raised to 90 billion dollars, 27 billion dollars of which were covered, agreeing with our assumption of stability regarding a 30% coverage. The southeastern area of the USA was touched by 6 wind events:
- An ice storm in Texas in February
- An ice storm in March 2015
- Three successful tornados in Oklahoma and Arkansas in March
- The storm “Bill” in Texas in June
- The hurricane “Joaquin” in October
- Tornadoes and storms in December

For 2015, our model forecasts 5 disasters between June and October:
- One in our learning base whose cost is up to 6 million dollars
- 4 in our forecast base whose cost are up to 1.3 billion dollars for three of them and 9.5 billion dollars for the other.

The model forecasts disasters during the most susceptible period of storms and hurricanes in the southeastern area of the USA. It simulates a high number of disasters close to the actual number. For the associated costs, it is hard to know if they are accurate. Indeed, most of the costs occurred during the year 2015 are not fully estimated yet. For instance, it is not possible to assess the damage of the tornados and storms that occurred in December 2015. For other disasters, estimations have been done. Regarding the hurricane “Joaquin” insured damages are between 25 and 30 million dollars. Considering the average rate of coverage in the world, which is around 30%, the cost of damages is up to 1 billion dollars approximately.
Nevertheless, the final goal of the model remains the pricing of insurance products. There are economic issues behind this method of pricing. Depending on the decisions of rating agencies and the investors’ behavior in the coming years, the cat bonds market could explode in volume. In this context, if investors are not able to assess properly the price of cat bond, a speculative bubble could arise. In the event of the occurrence of many disasters during a short period, this bubble could explode and will generate a ton of losses across financial markets. Finally, products that are supposed to be non-correlated with market changes could actually have an impact if an adverse scenario occurs.
Table of figures

Figure 1 Example of insurance risk transfer ................................................................. 5
Figure 2 Example of a 30XL20 contract ................................................................... 5
Figure 3 functioning of a cat bond ........................................................................... 7
Figure 4 Cat bond – Step by Step ........................................................................... 7
Figure 5 Differences between reinsurance and securitization of insurance risks .......... 8
Figure 6 Type of natural disasters in the world between 1990 and 2007 (source: http://www.notre-planete.info/terre/risques naturels/catastrophes naturelles.php)) ................................................................... 9
Figure 7 Distribution of cat bonds modeled by agencies (source: DUBREUIL E. [2010] “La titrisation des grands risques : Evolution ou Revolution ?”) ................................................................. 9
Figure 8 Number of disasters since 1950 (source: www.notre-planete.info/terre/risques naturels/catastrophes naturelles.php) ............................................................................................. 11
Figure 9 Cost of natural disasters since 1950 (source: www.notre-planete.info/terre/risques naturels/catastrophes naturelles.php) ............................................................................................. 11
Figure 10 Comparison of markets volumes (source: DUBREUIL E. [2010] ”La titrisation des grands risques : Evolution ou Revolution ?”) ................................................................. 12
Figure 11 Study area ................................................................................................. 14
Figure 12 Correlation circle ...................................................................................... 15
Figure 13 Annual costs according axis 1 and 2 .................................................................. 16
Figure 14 Artificial Neural Network - Multi Layer Perceptron – Regression model .................................................................................................................. 17
Figure 15 Illustration of a given path of an explanatory variable ................................. 17
Figure 16 Calibration methodology ......................................................................... 19
Figure 17 Results from the Test sub-base .................................................................. 20
Figure 18 Sensitivity – Case 1 .................................................................................. 21
Figure 19 Sensitivity – Case 2 .................................................................................. 22
Figure 20 Sensitivity – Case 3 .................................................................................. 22
Figure 21 Correlation circle – Axis 1 & 2 ................................................................... 23
Figure 22 Representation of catastrophes according axis 1 & 2 .................................. 23
Figure 23 Correlation circle – Axis 3 & 4 ................................................................... 24
Figure 24 Representation of catastrophes according axis 3 & 4 .................................. 24
Figure 25 Dependent variable according to the explanatory variables – Conventional case .......................................................... 25
Figure 26 Dependent variable according to the explanatory variables – Time delayed case .......................................................... 25
Figure 27 Forecast base in the case of a time delayed model ..................................... 25
Figure 28 Artificial Neural Network – Classification model ....................................... 26
Figure 29 Methodology for detecting catastrophes and distributing costs – Step 1 to 3 .......................................................... 27
Figure 30 Methodology for detecting catastrophes and distributing costs – Step 3 to 4 .......................................................... 28
Figure 31 Methodology for detecting catastrophes and distributing costs – Step 4 to 5 .......................................................... 28
Figure 32 Methodology for detecting catastrophes and distributing costs – Step 5 to 7 .......................................................... 29
Figure 33 Results from the Test sub-base – Classification model ................................ 29
Figure 34 Forecasted catastrophes 2015-2018 ............................................................ 30
Figure 35 Sensitivity – Case 1 .................................................................................. 31
Figure 36 Sensitivity – Case 1 .................................................................................. 31
Figure 37 Heritage Insurance Company’s main competitors and market share ............ 32
Figure 38 Heritage Insurance Company’s reinsurance program ................................ 32
Figure 39 Heritage Insurance Company’s reinsurance program – Contracts’ illustration .......................................................... 33
Figure 40 Example of coverage .............................................................................. 33
Figure 41 Premiums’ forecasts ................................................................................ 35
Figure 42 Pricing of the reinsurance program of Heritage Insurance Company .......... 35
Figure 43 Pricing of the reinsurance program of Heritage Insurance Company – Adjusted costs .......................................................... 36
Figure 44 Pricing of the reinsurance program of Heritage Insurance Company - Illustration .......................................................... 36
Figure 45 Ceded premiums by Heritage Insurance Company .................................... 36
Figure 46 Cat bonds’ coupons – Estimations vs. observations ................................... 38
Figure 47 Cat bond pricing using various methods ................................................... 38
Figure 48 Cat bond pricing taking into account the positive error ................................ 39
Figure 49 Range of coupons .................................................................................... 39
Figure 50 Real coupons and their estimated range ................................................... 39
Bibliography

Jemli R., Chtourou N., Feki R., Bazin D. [2012]

Charpentier A. [2002]
"Titrisation des risques catastrophes : les Cat-Bonds" Statistical study, French Federation of Insurance Companies

Chenal D., Kayo de kayo G., Kelhiouen R., Milhaud X., Sauser C. [2008]
"Titrisation du risque de catastrophe naturelle " Alternative risk transfer project supervised by Stéphane Loisel, M2R, SAS, ISFA.

Dubreuil E. [2010]
"La titrisation des grands risques : Evolution ou Révolution ?" Presentation, “Econometrics & Economics of insurance” day, “High Risks & Securitisation”

"Les nouvelles frontières de la réassurance" The Insurance interviews, Workshop 3, French Federation of Insurance Companies.

Kern E. [2014]
"Le marché de la titrisation en Europe : caractéristiques et perspectives” Analysis and Synthesis, ACPR, Banque de France

Besse P., Laurent B. [2014]
"Apprentissage Statistique : modélisation, prévision et data mining" Course, fifth year, INSA Toulouse.

Haffar A., Hikkerova L.[2014]
"Gestion et titrisation des risques de catastrophe naturelle par les options" Publication Working Paper, IPAG.

Mauroux A.[2015]
"Exposition aux risques naturels et marchés immobiliers." Quarterly review of the Association d'économie financière, n117, 91-103.

http://investors.heritagepci.com/index.php
Presentation, September 2015.

www.notre-planete.info/terre/risques naturels/catastrophes naturelles.php

Natural catastrophes’ data, July 2015.

www.ncdc.noaa.gov
Natural catastrophes’ data, July 2015.

http://www.artemis.bm/deal directory/
Cat bonds transactions’ data, September 2015.
Contacts

Benoît Genest | Partner | Head of Global Research & Analytics
bgenest@chappuishalder.com

Mikaël Benizri | Actuary
mbenizri@chappuishalder.com