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# **Model Risk Management | How to measure and quantify model risk?**

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## **Model Risk Management | How to measure and quantify model risk?**

### **Abstract**

In the context of an increasing number of models within banks, a need for more awareness regarding the risks stemming from their use has arisen. Indeed, one must always bear in mind that models and their outputs, as good and efficient as they can be, remain estimations and approximations of future events. Therefore, they must never be relied on as facts or values that will definitely happen. Hence, a model owner must always be aware of the different risks arising from the use of their models, either in terms of the models' inherent estimation errors or in terms of potential misuse of models and have an appropriate framework set up to manage these types of risks.

The aim of this paper is to present model risk situations and a methodology to measure and quantify the associated risk at model level, with different types of assumptions. Then, considering that in practice, a model risk management at model level is hardly feasible, this paper also outlines a method to measure and quantify model risk at risk category level (ex: Credit Risk).

In fact, one of the overarching drivers of this paper is to provide a model risk "value" which will enable you to analyse if the model risk is sufficiently covered. Indeed, although banks already allocate funds regarding this risk (portion of RWA attributed to conservative margins for credit risk, portion of Op risk Value at Risk, etc.), assessing the appropriateness of those funds remain complicated.

**Key words:** *Models, Risk Modeling, Model Risk Management (MRM), Credit Risk, Expected Loss (EL), Margin of Conservatism (MoC), Normal distribution, Central Limit Theorem, Markovian process*

**Classification JEL :** C13, C16, C41, C51, D81, G21

## Introduction

The increasing trend of using risk models within banks has led regulators to require more control over them. Either in Europe through the TRIM regulation (Target Review of Internal Models) required by the EBA or in the US through SR 11-7 or new qualitative requirements within stress-testing exercises, all banks must guarantee an efficient management of the extending use of models. This is especially important since the scope of models under supervision continues to broaden over time. ALM, Liquidity, non-financial models, credit scores for loan granting, etc. are to be added to Pillar I models (Credit risk, Market risk and Operational risk), to become part of a consolidated management for all financial companies.

Considering this “huge inventory“, most banks define two types of models that could lead to two different types of risk:

- **Regulatory approved models** | Regarding this type of model built under regulatory constraints, risk arises from an underestimation of the forecasted losses against the realised ones. Theoretically speaking, it means that banks must be able to cover this specific risk with their own funds (capital adequacy). In practice, even though first attempts of Pillar II measures do exist, they remain not quite as advanced.
- **Decision-making models** | For this second type of model (loan granting credit scores, product pricing, etc.), it is more a misuse or an incorrect implementation that is being highlighted. As such, this type of model risk is supposed to be considered within Operational risk. New Basel III guidelines in this area (SMA) are therefore supposed to cover these risks, while not necessary isolating or quantify them solely.

It is interesting to note that depending on the group to which the models belong, the risk coverage is not performed in the same way. Indeed, if capital raised by banks as part of the operational risk enables the coverage of a misuse or a poor implementation of a model, it is not the same for models of the first group.

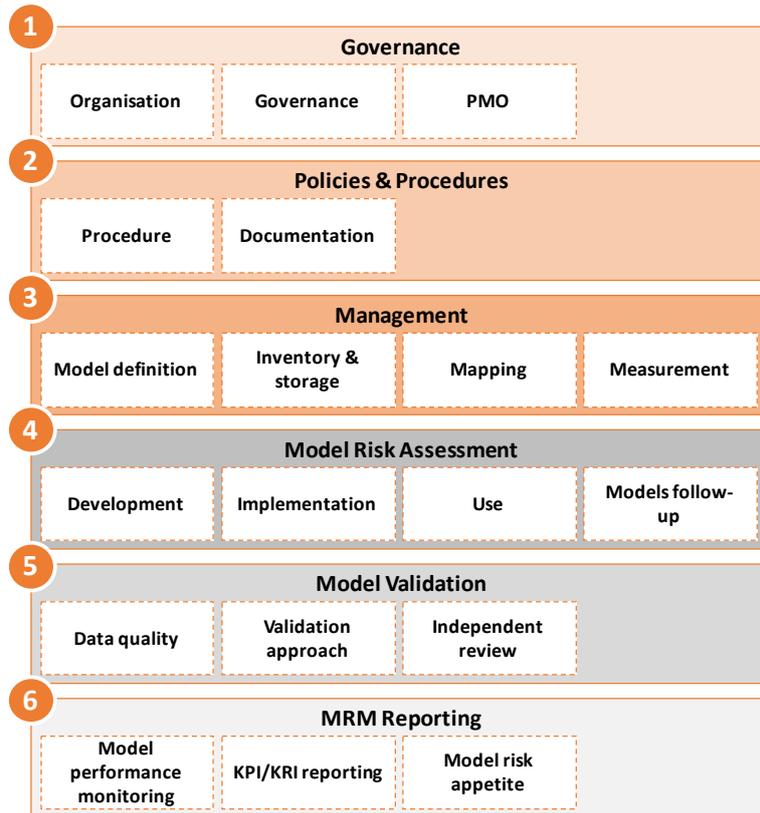
Regarding the regulatory approved models, banks must:

- Define model risk;
- Measure model risk;
- Estimate an amount of capital charge to cover this risk as part of Basel Pillar II.

These three steps are still at an early stage. The definition of a model, their inventory, the relatively recentness of the corresponding regulatory texts and the onsite audits performed by regulatory authorities on this topic, explain the not so advanced stage as it remains relatively new.

The intent of this paper is to analyse how model risk management requirements change the banks' view on their models, especially regarding the quantification of associated risks, and to introduce a new framework methodology. It would provide first answers to the requirements of European regulators regarding model risk management. In this context, one can note the maturity gap between the management of the risk stemming from models and the measurement of model risk. Despite banks have taken charge of the management of risk models during the last year in order to control them as part of their daily business, it is not quite the same regarding the measurement of model risk.

In fact, the measurement of model risk is one of the last components of the wider model management plan still in progress. Currently, few methodologies already exist or seem suited to properly value the risk related to the use of models. Figure 1 below illustrates the main components of the model management framework of most banks. Indeed, they all drew the main design of their framework from the principles of the North America RCO council (August 2016) and from the Fed supervisory guidelines on risk management (April 2011).



**Figure 1 | Model risk Management (MRM) framework**

*Source: CH&Co GRA*

Maybe one of the most crucial components remains the one regarding the quantification of the risk generated using models. The main unknown factor as of now is the inability to provide a “value” corresponding to their MRM. Therefore, we decided to conduct research specific to this component within this paper. Indeed, while banks start to list, to verify and to communicate regarding their models, they now have to answer the questions:

***“What is the value of your model risk? Is it properly covered?”***

To sum up, this paper initially aims to provide suggestions on how to answer those questions.

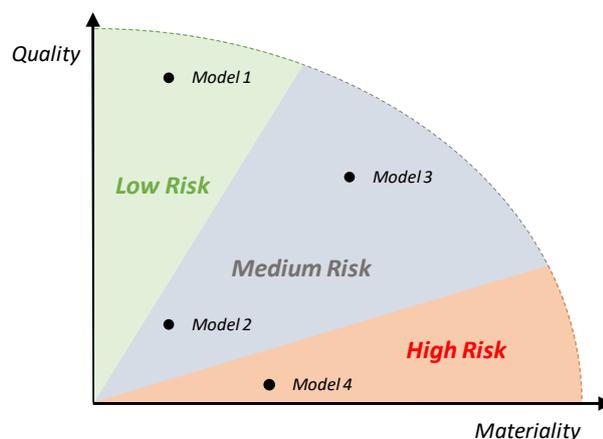
In this context, the measurement and the follow-up of the banking model risk become one of the main current challenges. Especially considering the past loss events stemming from a misuse of models (subprime, rating of money lines by rating agencies models, pricing errors on the markets, etc.). Nowadays, financial companies have hundreds of models, even thousands of models for some of them. Therefore, the Regulator cannot afford to let the potential risk of using those models not well understood or not properly covered anymore.

The paper is structured in two sections. The first one describes a methodology to quantify the model risk at model level. In other words, it lays the foundation for the estimation of an idiosyncratic and individual risk per model. The second section however, aims at proposing a method to measure model risk at risk type level. This latest measure could thus be used to challenge current model risk funds and/or to compute the potential additional capital that banks would have to raise as part of Pillar II.

## 1. Overview

As of now, first approaches are implemented by banks regarding the “Model Risk Assessment” that consist in evaluating the level of risk based on objective criteria. Most of them generally cover two main axes, the first one being the level of materiality (impact concept) and the second one the level of quality (concept of occurrence probability of this impact). Those approaches usually result in:

- **A scorecard approach** (continuous version): it enables the Bank to compute a weighted grade which is, for instance, compared to the mean of similar models or over a specific perimeter (subsidiaries for example), and to benchmark the models, as well as follow the grades historical movements.
- **A mapping approach** (discrete version): this approach locates the models using the two axes and needs to define areas on which the models are considered “low risk”, “medium risk” and “high risk”, as illustrated on the Figure 2 below.



**Figure 2 | Model risk assessment – Discrete approach**

*Source: CH&Co GRA*

Even though it slightly changes from one bank to another, these approaches are mainly built by assessing the quality/risk of the following components:

- Model Misuse;
- Conceptual soundness;
- Model misspecifications;
- Estimation uncertainty & Initial condition uncertainty;
- Parameter changes & Variable mismeasurements;
- Errors in development & Errors accumulation.
- Missing risk factors & Missing considerations;
- Etc.

The use of similar components among banks can be explained by regulatory requirements but also by the quality of the frameworks and systems currently in place. Causes of risks are now well known and identified. They are also considered within the processes of risk monitoring within banking institutions. These approaches are performed at model level. As explained, their main purpose is to rank the models and to compare them. This is relevant and useful, especially when it comes to identifying the need for further development or to prioritise work such as model reviews for instance. In other words, leveraging on the outcomes of this type of analysis enables a more efficient building of the book of work for modelling and validation teams and to manage the resources and efforts needed accordingly. Despite those advantages, these approaches remain causal, meaning that they explain the level of risk based on

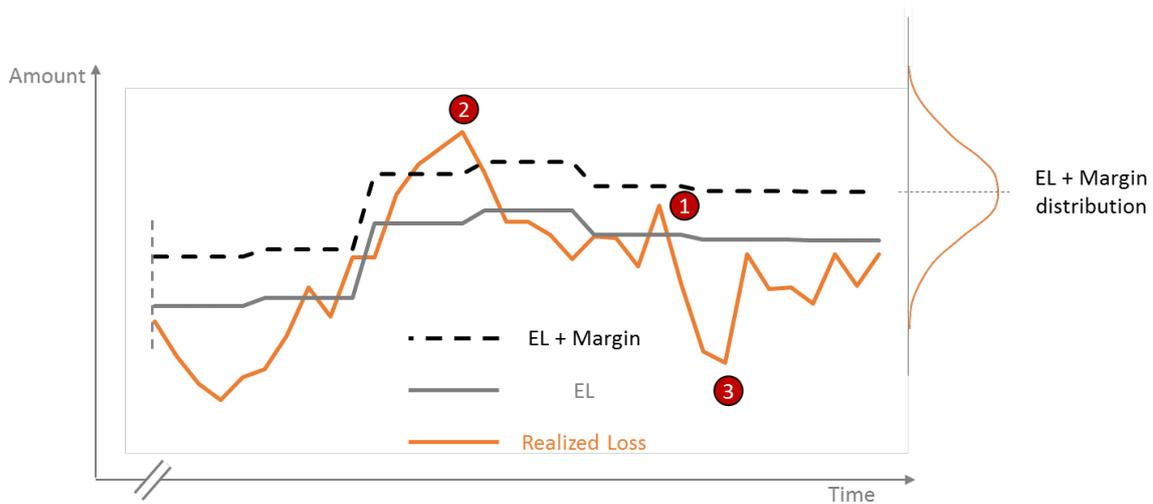
its sources but do not provide a value of model risk. Therefore, they need to be supplemented by a measurement enabling the quantification of this risk, not only to rank them.

Within this paper, our work consists in the designing of a non-causal global methodology. More precisely, it means that the measurement does not rely on the causes of occurrence of the risk. Since Model Risk Management encompasses all models of a bank, the overarching concept is to build a measurement that can be applied to all types of models.

Therefore, the starting point of this paper is to consider that a model risk can be defined by relying only on the gaps observed between the forecasted values based on the outcomes of the model and the realised values. The considered methodology thus consists of analysing the distribution of those gaps and to deduce a measurement of model risk based on this distribution.

## 2. What we define as Model Risk situations

Within this paper, we chose to focus on three types of model risk situations, all based on the differences that can be observed between the Expected Loss and the realised ones. Figure 3 below displays the three situations considered herein:



**Figure 3 | Model Risk situations**

*Source: CH&Co GRA*

**Situation 1** | An underestimation of the realised losses, but that remain covered by the inclusion of the margin of conservatism (MoC). One could argue that this situation does not generate a model risk since the realised losses are covered. However, the information regarding the margin of conservatism’s “consumption level” is fundamental in setting up a prospective model risk management framework and to avoid the necessity of suffering an uncovered loss to react and plan remediations. Therefore, we chose to consider this case as part of the Model Risk situations. This situation is denoted “Model Risk 1” throughout the rest of this paper.

**Situation 2** | An underestimation of the realised losses, larger than the margin of conservatism and therefore not covered. This situation is the basic and minimum one to consider since it corresponds to an actual loss. Preventing this situation represents the main purpose of a Model Risk Management framework and the main reason that has led to the Model Risk Management requirements. This situation is denoted “Model Risk 2” throughout the rest of this paper.

**Situation 3** | A large overestimation of the realised losses. Once more, as in the first situation described above, one could argue that this situation does not correspond to a model risk one. However, even though the risk of loss stemming from an underestimation is unlikely to occur in this case, such inaccurate

results could lead to misjudgement or poor decision making. Although being conservative, it could lead to other types of losses. One should bear in mind that a model that strongly overestimates the loss it is supposed to model, if not being well understood, next time could lead to a strong underestimation of the loss. Indeed, if a model is not fit for purpose, its future results are unpredictable, even if they could seem conservative at a specific moment. This situation is denoted “Model Risk 3” throughout the rest of this paper.

### 3. Expected Loss & Errors distributions

Depending on the model type analysed, a “loss” can be defined in different ways. Indeed, when considering risk models such as IRB Credit risk, IFRS9 Credit risk, Market risk, Counterparty risk or Operational risk models for instance, the loss definition is quite straightforward. However, for other models used by banks such as marketing models, decision making models or even pricing models, the loss can correspond to different types of situations. However, within this paper, the focus is made on risk models. Regardless of the definition, to cover future losses, an estimator is computed. It is theoretically based on historical data in such way that the following equation is obtained:

$$\begin{aligned} \mathbb{E}(\text{Realized Loss}_T - \text{Expected Loss}_T | \mathcal{F}_{T-1}) &= 0 \\ \Leftrightarrow \text{Expected Loss}_T &= \mathbb{E}(\text{Realized Loss}_T | \mathcal{F}_{T-1}) \end{aligned}$$

The underlying meaning is that, on average, the realised loss of the following period should be equal to the estimator computed at the beginning of the period (*Expected Loss<sub>T</sub>*). To measure model risk based on the behaviour of this estimator and the corresponding realised values, an analysis of the gaps between them is performed.

#### 3.1. Expected Loss estimator

It is commonly assumed that yearly realised losses are independent and identically distributed. Therefore, based on the law of large numbers, one can obtain the following convergence:

$$\frac{1}{T-1} \sum_{i=1}^{T-1} \text{Realized Loss}_i \xrightarrow{T} \mathbb{E}(\text{Realized Loss})$$

And the estimation of the realized loss that will occur during the time period T then corresponds to  $\mathbb{E}(\text{Realized Loss})$ .

$$\mathbb{E}(\text{Realized Loss}_T | \mathcal{F}_{T-1}) = \mathbb{E}(\text{Realized Loss})$$

In practice, the estimator is almost never computed directly using the amounts of realized losses. For instance, if we consider a Credit risk portfolio or counterparty, the estimator of the corresponding future losses (Expected Loss) would be computed as follow:

$$EL = \mathbb{E}(\text{Realized Loss}) = PD * LGD * EAD$$

Where *PD* corresponds to the Probability of Default parameter, *LGD* to the Loss Given Default parameter (in percentage) and *EAD* to the Exposure at Default. Even though not computed directly using the realised losses, it should converge towards the same value.

#### 3.2. Expected Loss errors distribution

In order to properly use the distribution features of the gaps, one must consider the gaps between the loss estimator and the occurred loss in percentage of the losses initially estimated (Expected Losses), in order to avoid biases coming from level effects:

$$(D_k)_{k \in \{1, \dots, T\}} = \left( \frac{\text{Expected Loss}_k - \text{Realised Loss}_k}{\text{Expected Loss}_k} \right)_{k \in \{1, \dots, T\}}$$

With *D* assumed to be a random variable following a normal distribution (Central Limit Theorem), representing the distance between the realised loss and its estimator, in proportion of this estimator:

$$D \sim \mathcal{N}(\mathbb{E}(D), \sigma_D^2)$$

And since

$$\begin{aligned} \mathbb{E}(D) &= \mathbb{E}\left(\frac{\text{Expected Loss} - \text{Realised Loss}}{\text{Expected Loss}}\right) \\ &= 1 - \mathbb{E}\left(\frac{\text{Realised Loss}}{\mathbb{E}(\text{Realised Loss})}\right) \\ &= 1 - \frac{1}{\mathbb{E}(\text{Realised Loss})} * \mathbb{E}(\text{Realised Loss}) \\ &= 0 \end{aligned}$$

One can get  $D \sim \mathcal{N}(0, \sigma_D^2)$ , and  $(D_k)_{k \in \{1, \dots, T\}}$  corresponds to the observations of this random variable over T time periods.

In practice, when estimating the Expected Loss, a prudence margin is usually added to the initial estimation to obtain a conservative estimator which allows one to reduce the risk of underestimation of the loss. Indeed, since the Expected Loss estimator corresponds to an average value, adding a margin to take into account the volatility of the estimator is good practice (Credit Risk “C Margin”). Also, other types of margins are commonly added in order to consider the known weaknesses of the model (poor data quality or impacts of significant changes for instance – Credit Risk “A, B Margins”). However, as explained earlier with the example of an Expected Loss computed for a Credit risk portfolio, the estimation is performed using three parameters ( $PD$ ,  $LGD$  and  $EAD$ ) and not directly realised losses. It goes the same way regarding the conservative margins, which are applied at parameter level. For example, the margin of conservative aiming to cover the volatility of the PD parameter (“C Margin”) can be computed as the  $\alpha\%$  upper bound of the one-sided confidence interval of the default rates, resulting in the following:

$$\text{Margin}_{PD} = q_{1-\alpha\%} * \sigma_{PD}$$

With  $q_\alpha$  the  $\alpha\%$  quantile of a standard normal distribution  $\mathcal{N}(0, 1)$ .

Once all the margins involved in the computation of the Expected Loss are considered, the estimator can be rewritten as follow:

$$\text{Expected Loss}_T^{\text{Margin}} = \widehat{\mathbb{E}}(\text{Realised Loss}) + \widehat{M}_{\text{Amount}}$$

With  $\widehat{M}_{\text{Amount}}$  corresponding to the estimation of the total conservative margin.

Hence, the difference between the forecasted loss and the realised one can also be rewritten as below:

$$D^M = \frac{\text{Expected Loss}^{\text{Margin}} - \text{Realised Loss}}{\text{Expected Loss}^{\text{Margin}}}$$

And the expected value of  $D^M$  becomes:

$$\begin{aligned} \mathbb{E}(D^M) &= \mathbb{E}\left(\frac{\text{Expected Loss}^{\text{Margin}} - \text{Realised Loss}}{\text{Expected Loss}^{\text{Margin}}}\right) \\ &= \frac{\widehat{M}_{\text{Amount}}}{\text{Expected Loss}^{\text{Margin}}} \\ &= \widehat{M} \geq 0 \end{aligned}$$

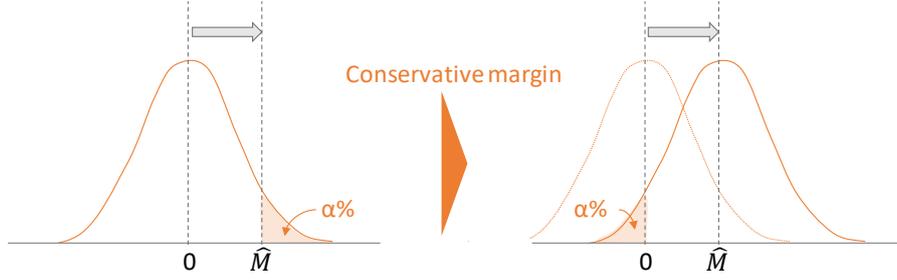
Giving a normal distribution with adjusted parameters:

$$D^M \sim \mathcal{N}(\widehat{M}, \sigma_{D^M}^2)$$

Another way to formulate  $D_t^M$  would be:

$$\begin{cases} D_t^M = \widehat{M} + \sigma_{D^M} * \xi \\ \xi \sim \mathcal{N}(0, 1) \end{cases}$$

If one assumes  $\hat{M} = q_{1-\alpha\%} * \sigma_{DM}$ , the distribution of the gaps, with and without the inclusion of the overall margin of conservatism can be illustrated as below:



**Figure 4 | Gaps' distribution with and without conservative margin**

Source: CH&Co GRA

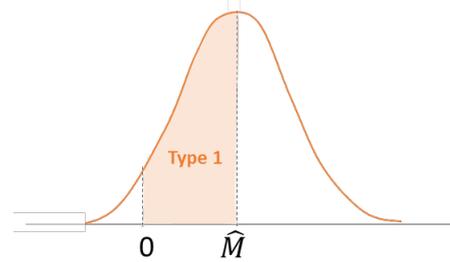
#### 4. Probability of occurrence of the different model risk situations

The following subsections derive the probability of occurrence of each of the scenarios introduced earlier by relying on the distribution of gaps defined in the previous section.

##### Model Risk 1

The idea is to answer the question “*What is the expectation of being covered, thanks to the prudence margin, but not by the initial expected loss, one can have using the model?*”. This first situation corresponds to an underestimation of the realised loss by the expected loss estimator but offset by the conservative margin that has been added to it. Therefore, by relying on the distribution  $D^M \sim \mathcal{N}(\hat{M}, \sigma_{DM}^2)$ , the expectation of occurrence of such a situation can be expressed as follow:

$$\begin{aligned}
 \text{ModRisk}_1 &= \mathbb{E}(\mathbb{1}_{\{0 \leq D^M \leq \hat{M}\}}) \\
 &= \mathbb{P}(\{0 \leq D^M \leq \hat{M}\}) \\
 &= \mathbb{P}(\{D^M \leq \hat{M}\}) - \mathbb{P}(\{D^M \leq 0\}) \\
 &= \frac{1}{2} - \mathbb{P}(\{D^M \leq 0\}) \\
 &= \frac{1}{2} - \mathbb{P}\left(\left\{\xi \leq \frac{-\hat{M}}{\sigma_{DM}}\right\}\right) \\
 &= \frac{1}{2} - \left(1 - N\left(\frac{\hat{M}}{\sigma_{DM}}\right)\right) \\
 &= N\left(\frac{\hat{M}}{\sigma_{DM}}\right) - \frac{1}{2}
 \end{aligned}$$



**Figure 5 | Model Risk – Type 1**

Source: CH&Co GRA

With

$\xi = \text{random variable following a standard normal distribution } \xi \sim \mathcal{N}(0, 1)$   
 $N(\cdot) = \text{cumulative distribution function of a standard normal distribution}$

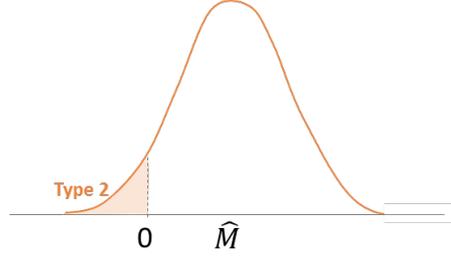
In addition, if the conservative margin is  $\hat{M} = q_{1-\alpha\%} * \sigma_{DM}$ , the following result is obtained:

$$\begin{aligned}
 \text{ModRisk}_1 &= N\left(\frac{\hat{M}}{\sigma_{DM}}\right) - \frac{1}{2} \\
 &= N(q_{1-\alpha\%}) - \frac{1}{2} \\
 &= \frac{1}{2} - \alpha\%
 \end{aligned}$$

### Model Risk 2

The idea is to answer the question “*What is the expectation of not being covered enough, even considering the prudence margin, one can have using the model?*”. This second situation corresponds to a more severe case, when the expected loss including the conservative margin is exceeded by the realised loss. In other words, it corresponds to an actual loss from a risk point of view. Therefore, by relying on the distribution  $D^M \sim \mathcal{N}(\hat{M}, \sigma_{D^M}^2)$ , the expectation of occurrence of such a situation can be expressed as follow:

$$\begin{aligned}
 \mathbf{ModRisk}_2 &= \mathbb{E} \left( \mathbb{1}_{\{D^M \leq 0\}} \right) \\
 &= \mathbb{P}(\{D^M \leq 0\}) \\
 &= \mathbb{P}(\{\hat{M} + \sigma_{D^M} * \xi \leq 0\}) \\
 &= N \left( \frac{-\hat{M}}{\sigma_{D^M}} \right) \\
 &= 1 - N \left( \frac{\hat{M}}{\sigma_{D^M}} \right)
 \end{aligned}$$



**Figure 6 | Model Risk – Type 2**

Source: CH&Co GRA

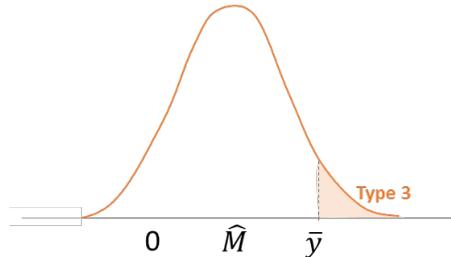
In the same vein, if the margin of conservative corresponds to  $\hat{M} = q_{1-\alpha\%} * \sigma_{D^M}$ , then the following results can be obtained:

$$\begin{aligned}
 \mathbf{ModRisk}_2 &= 1 - N \left( \frac{\hat{M}}{\sigma_{D^M}} \right) \\
 &= 1 - N \left( \frac{q_{1-\alpha\%} * \sigma_{D^M}}{\sigma_{D^M}} \right) \\
 &= 1 - N(q_{1-\alpha\%}) \\
 &= 1 - (1 - \alpha\%) \\
 &= \alpha\%
 \end{aligned}$$

### Model Risk 3

The idea is to answer the question “*What is the expectation of not being accurate enough, even in a conservative way, one can have using the model?*”. Finally, this third case corresponds to the risk of a significant lack of accuracy from the model, leading to overly conservative and un-reliable results, particularly for business use. It is considered good practice to define limits that should not be exceeded to consider the model accurate enough. Regarding this model risk situation, it is a downward limit that we are interested in (see Figure 3), that is denoted  $\bar{y}$  throughout the rest of this paper. Once more, by relying on  $D^M \sim \mathcal{N}(\hat{M}, \sigma_{D^M}^2)$ , the expectation of occurrence of such a situation can be expressed as follow:

$$\begin{aligned}
 \mathbf{ModRisk}_3 &= \mathbb{E} \left( \mathbb{1}_{\{D^M > \bar{y}\}} \right) \\
 &= \mathbb{P}(\{D^M > \bar{y}\}) \\
 &= 1 - \mathbb{P}(\{D^M \leq \bar{y}\}) \\
 &= 1 - N \left( \frac{\bar{y} - \hat{M}}{\sigma_{D^M}} \right)
 \end{aligned}$$



**Figure 7 | Model Risk – Type 3**

Source: CH&Co GRA

Hence, relying on the distribution of errors enables us to derive simple formulas to compute the probabilities of occurrence of the three model risk situations considered here. However, and especially in the context of model risk, one must consider the fact that the distribution of errors is based on assumptions. Since one of the main axes of the Model Risk Management philosophy is to assess the potential impacts of inappropriate assumptions of the models used by banks, it is therefore necessary to consider those within a methodology contemplated to measure model risk. As explained in section 3.2, the main assumptions used to derive the convergence and the normal distribution are:

- Identically distributed variable;
- Independently distributed variable.

The next section focuses on these two assumptions.

## 5. Inclusion of complementary features to the assumed normal distribution

The assumption of normal distribution is not questioned within this paper. Nevertheless, as a first step, its features are questioned. In other words, the mean and the standard deviation of a normal distribution are the two parameters that determine the shape of the normal distribution. When using the central-limit theorem, these two parameters are considered steady over time, stemming from the identical and independent distribution of the variable considered. Therefore, the steadiness of the distribution parameters is called into question. The theoretical examples provided within the rest of this paper focus on the mean parameter but could easily be extended to the standard deviation one. Two types of impact are considered, the dependency towards external economic factors (questioning the identical distribution of the variable through years) as well as the dependency to the last observed value (questioning the independency and the identical distribution of the variable).

Remaining based on the normal distribution of the residuals, the general distribution formula of  $D^M$  considering those aspects can be expressed as below:

$$D_t^M = f\left(\widehat{M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}\right) + g\left(\sigma_{D^M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}\right) * \xi$$

With

- $f\left(\widehat{M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}\right)$  corresponding to the mean parameter, that depends on the margin of conservatism ( $\widehat{M}$ ), and potentially to the last value observed ( $D_{t-1}^M$ ) as well as external variables ( $E_{t-1,j}$ ).
- $g\left(\sigma_{D^M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}\right)$  corresponding to the standard deviation parameter, depending on the theoretical initial estimation of the standard deviation ( $\sigma_{D^M}$ ), and also potentially to the last value observed ( $D_{t-1}^M$ ) as well as external variables ( $E_{t-1,j}$ ).
- $\xi$  remains a random variable following a standard normal distribution  $\xi \sim \mathcal{N}(0, 1)$ .

In the previous section, it is assumed that both parameters, mean and standard deviation, are steady and their level depends only on the margin of conservatism for the mean parameter and the variance of the errors for the standard deviation one. Using the general formulas introduced above, one can deduce the following:

$$\begin{aligned} f\left(\widehat{M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}\right) &= f(\widehat{M}) = \widehat{M} \\ g\left(\sigma_{D^M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}\right) &= g(\sigma_{D^M}) = \sigma_{D^M} \end{aligned}$$

$D_t^M$  can then be expressed as follows:

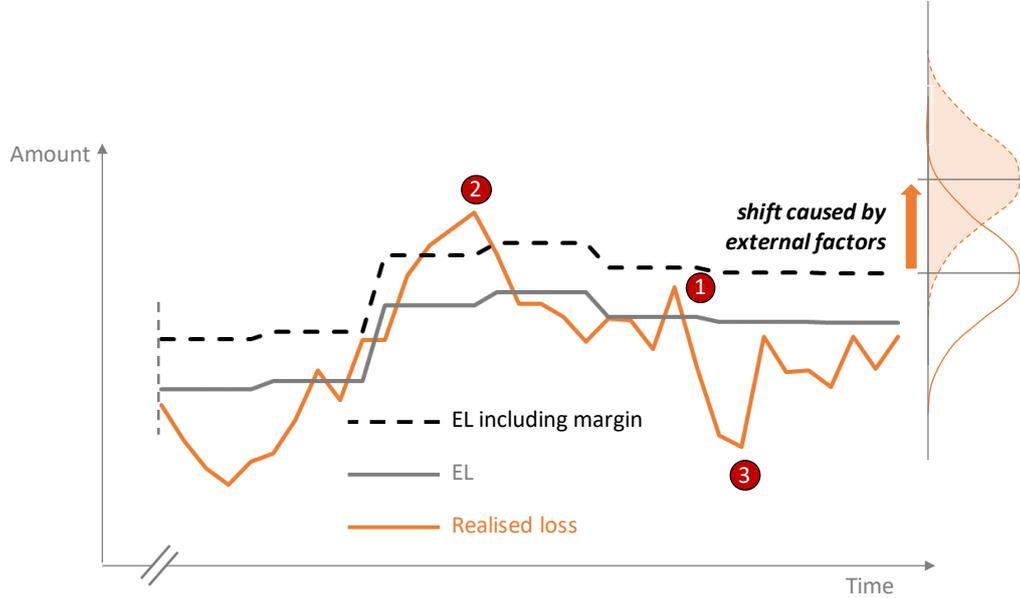
$$D_t^M = \widehat{M} + \sigma_{D^M} * \xi$$

It enables us to find the distribution as expressed earlier:

$$D_t^M \sim \mathcal{N}(\widehat{M}, \sigma_{D^M}^2)$$

## 5.1. External factors dependency

Theoretically speaking, the gaps are identically and independently distributed over time. However, in practice, external factors (or events) such as economic movements can impact them. One can consider that it represents model risk stemming from the systemic risk. Recall that as an example, only the mean parameter is considered impacted in this paper, as illustrated on Figure 8 below.



**Figure 8 | Potential impacts on the distribution caused by external factors**

Source: CH&Co GRA

Therefore, using the formulas introduced, one can express the mean parameter as follow:

$$f(\hat{M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}) = f(\hat{M}, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}) = \hat{M} + \sum_{j=1}^J \varphi_j * E_{t-1,j}$$

Where  $\varphi_j$  corresponds to the weight of impact associated to the «  $j^{th}$  » external factor level  $E_{t-1,j}$  as of the date of estimation «  $t - 1$  » and,

$$g(\sigma_{D^M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}) = g(\sigma_{D^M}) = \sigma_{D^M}$$

$D_t^M$  can be expressed as follow:

$$D_t^M = \hat{M} + \sum_{j=1}^J \varphi_j * E_{t-1,j} + \sigma_{D^M} * \xi$$

$$D_t^M = \underbrace{\hat{M} + \sigma_{D^M} * \xi}_{\text{Inherent in the model}} + \underbrace{\sum_{j=1}^J \varphi_j * E_{t-1,j}}_{\text{Shift caused by external factors}}$$

And,

$$D_t^M \sim \mathcal{N}\left(\hat{M} + \sum_{j=1}^J \varphi_j * E_{t-1,j}, \sigma_{D^M}^2\right)$$

Now that the adjusted distribution is obtained, one can recompute the probabilities of occurrence of the three model risk situations introduced within the previous section and obtain the following results:

$$\begin{aligned} \text{ModRisk}_1^{\text{Systemic}} &= N\left(\frac{\widehat{M} + \sum_{j=1}^J \varphi_j * E_{t-1,j}}{\sigma_{D^M}}\right) - N\left(\frac{\sum_{j=1}^J \varphi_j * E_{t-1,j}}{\sigma_{D^M}}\right) \\ \text{ModRisk}_2^{\text{Systemic}} &= 1 - N\left(\frac{\widehat{M} + \sum_{j=1}^J \varphi_j * E_{t-1,j}}{\sigma_{D^M}}\right) \\ \text{ModRisk}_3^{\text{Systemic}} &= 1 - N\left(\frac{\bar{y} - (\widehat{M} + \sum_{j=1}^J \varphi_j * E_{t-1,j})}{\sigma_{D^M}}\right) \end{aligned}$$

## 5.2. Past values dependency

Theoretically, gaps are considered independent over time. However, in practice, one can consider that gaps are not fully independent from one time period to another. Within this paper, the Markovian process case is considered, where the dependency on past variables is limited to the last observed value:

$$\mathbb{P}(\{X_T = x \mid X_{T-1}, X_{T-2}, \dots, X_0\}) = \mathbb{P}(\{X_T = x \mid X_{T-1}\})$$

Therefore, since it is considered that only the mean parameter is impacted by the last observed gap, one can express the parameters as follow:

$$\begin{aligned} f(\widehat{M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}) &= f(\widehat{M}, D_{t-1}^M) \\ g(\sigma_{D^M}, D_{t-1}^M, (E_{t-1,j})_{j \in \llbracket 1, J \rrbracket}) &= g(\sigma_{D^M}) \end{aligned}$$

And thus  $D_t^M$  as below:

$$D_t^M = f(\widehat{M}, D_{t-1}^M) + g(\sigma_{D^M}) * \xi$$

One can assume that the gap diffusion process can be modeled by an Ornstein–Uhlenbeck stochastic process such as the Vasicek model:

$$\partial D_t^M = \lambda (\widehat{M} - D_t^M) \partial t + \sigma_{D^M} \partial W_t$$

With

$$\partial W_t \sim \mathcal{N}(0, \partial t)$$

Where

$D_t^M$  = gap observed in  $t$

$\lambda$  = speed of mean reversion

$\widehat{M}$  = long – term mean level

$\sigma_{D^M}$  = volatility

$\partial W_t$  = movement of a standard Brownian motion over time period " $t$ "

Indeed, using this type of process, the increase of the uncertainty over the time horizon of estimations can be considered within the standard deviation of the Brownian motion. Moreover, it enables us to consider the situation at the time of measurement (last observed value), whereas it has no impact in the case where realised losses are considered independent.

Using the Itô lemma with the function  $f(t, D_t^M) = D_t^M e^{\lambda t}$ , one can obtain:

$$\begin{aligned} \partial f(t, D_t^M) &= \frac{\partial f(t, D_t^M)}{\partial t} \partial t + \frac{\partial f(t, D_t^M)}{\partial D_t^M} \partial D_t^M + \frac{1}{2} \frac{\partial^2 f(t, D_t^M)}{\partial D_t^{M^2}} \partial \langle D^M, D^M \rangle_t \\ \partial (D_t^M e^{\lambda t}) &= \lambda D_t^M e^{\lambda t} \partial t + e^{\lambda t} \partial D_t^M + \frac{1}{2} * 0 * \partial \langle D, D \rangle_t \\ \partial (D_t^M e^{\lambda t}) &= \lambda D_t^M e^{\lambda t} \partial t + e^{\lambda t} (\lambda (\widehat{M} - D_t^M) \partial t + \sigma_{D^M} \partial W_t) \end{aligned}$$

$$\begin{aligned} \partial(D_t^M e^{\lambda t}) &= e^{\lambda t} \lambda \widehat{M} \partial t + \sigma_{DM} e^{\lambda t} \partial W_t \\ \int_0^t \partial(D_s^M e^{\lambda s}) &= \int_0^t e^{\lambda s} \lambda \widehat{M} \partial s + \int_0^t \sigma_{DM} e^{\lambda s} \partial W_s \\ D_t^M e^{\lambda t} - D_0^M &= \widehat{M}(e^{\lambda t} - 1) + \int_0^t \sigma_{DM} e^{\lambda s} \partial W_s \\ D_t^M &= e^{-\lambda t} D_0^M + \widehat{M}(1 - e^{-\lambda t}) + \int_0^t \sigma_{DM} e^{-\lambda(t-s)} \partial W_s \end{aligned}$$

The mean and standard deviation can then be derived as follow:

$$\begin{aligned} f(\widehat{M}, D_0^M, \lambda) &= e^{-\lambda t} D_0^M + \widehat{M}(1 - e^{-\lambda t}) + \mathbb{E} \left( \int_0^t \sigma_{DM} e^{-\lambda(t-s)} \partial W_s \right) \\ &= e^{-\lambda t} D_0^M + \widehat{M}(1 - e^{-\lambda t}) \end{aligned}$$

and

$$\begin{aligned} g(\sigma_{DM}, \lambda) &= \sqrt{\mathbb{E} \left( \left( \int_0^t \sigma_{DM} e^{-\lambda(t-s)} \partial W_s \right)^2 \right)} \\ &= \sqrt{\mathbb{E} \left( \int_0^t \sigma_{DM}^2 e^{-2\lambda(t-s)} \partial s \right)} \\ &= \sqrt{\frac{\sigma_{DM}^2}{2\lambda} (1 - e^{-2\lambda t})} \end{aligned}$$

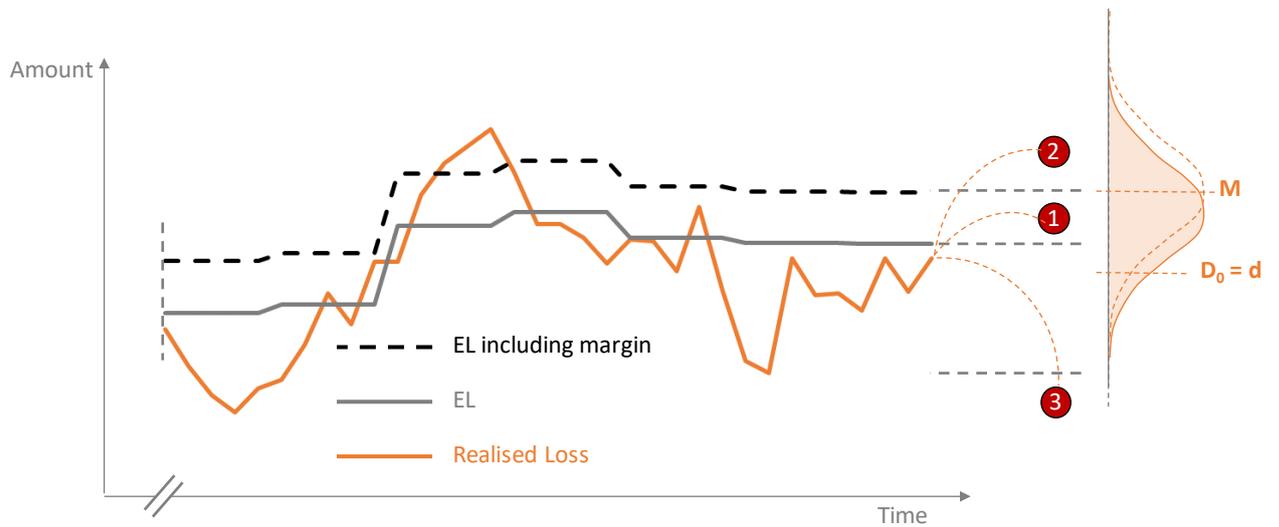
One can notice that the mean parameter corresponds to a weighted average of the long term mean of gaps and the last observed gap. The weighting depending on the speed of mean reversion. The higher this speed is, the closer the mean parameter is to the long term mean  $\widehat{M}$ . Using those parameters,  $D_t^M$  can be expressed as follow:

$$D_t^M = e^{-\lambda t} D_0^M + \widehat{M}(1 - e^{-\lambda t}) + \sqrt{\frac{\sigma_{DM}^2}{2\lambda} (1 - e^{-2\lambda t})} * \xi$$

And

$$D_t^M \sim \mathcal{N} \left( e^{-\lambda t} D_0^M + \widehat{M}(1 - e^{-\lambda t}); \frac{\sigma_{DM}^2}{2\lambda} (1 - e^{-2\lambda t}) \right)$$

By relying on this type of Markovian process, estimating the probability of reaching the different thresholds defined earlier (model risk situations) considering the impact of the last observed value ( $D_0^M = d$ ) is then doable.



**Figure 9 | Potential change on the distribution depending on last observed value**

Source: CH&Co GRA

Once the different parameters are estimated ( $\lambda, \sigma$ ), the following results can then be obtained:

$$\begin{aligned}
 \mathbf{ModRisk}_1^d &= \mathbb{E} \left( \mathbb{1}_{\{0 \leq D_t^M \leq \hat{M}\} | D_0 = d} \right) \\
 &= \mathbb{P}(\{0 \leq D_t^M \leq \hat{M} | D_0 = d\}) \\
 &= \mathbb{P}(\{D_t^M \leq \hat{M} | D_0 = d\}) - \mathbb{P}(\{D_t^M \leq 0 | D_0 = d\}) \\
 &= \mathbf{N} \left( \frac{\hat{M} - (e^{-\lambda t} d + \hat{M}(1 - e^{-\lambda t}))}{\frac{\sigma^2}{2\lambda} (1 - e^{-2\lambda t})} \right) - \left( 1 - \mathbf{N} \left( \frac{e^{-\lambda t} d + \hat{M}(1 - e^{-\lambda t})}{\frac{\sigma^2}{2\lambda} (1 - e^{-2\lambda t})} \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ModRisk}_2^d &= \mathbb{E} \left( \mathbb{1}_{\{D_t^M \leq 0\} | D_0 = d} \right) \\
 &= \mathbb{P}(\{D_t^M \leq 0 | D_0 = d\}) \\
 &= 1 - \mathbf{N} \left( \frac{e^{-\lambda t} d + \hat{M}(1 - e^{-\lambda t})}{\frac{\sigma^2}{2\lambda} (1 - e^{-2\lambda t})} \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{ModRisk}_3^d &= \mathbb{E} \left( \mathbb{1}_{\{D_t^M > \bar{y}\} | D_0 = d} \right) \\
 &= \mathbb{P}(\{D_t^M > \bar{y} | D_0 = d\}) \\
 &= 1 - \mathbb{P}(\{D_t^M \leq \bar{y} | D_0 = d\}) \\
 &= 1 - \mathbf{N} \left( \frac{\bar{y} - (e^{-\lambda t} d + \hat{M}(1 - e^{-\lambda t}))}{\frac{\sigma^2}{2\lambda} (1 - e^{-2\lambda t})} \right)
 \end{aligned}$$

Within this section, the objective was first to analyse model risk situations and to build first indicators enabling us to start measuring them. Also, the main assumptions of the Expected Loss error distribution have been questioned and first suggestions to address them have been provided.

The intent is not to provide what one could call the “right” method to perform the measurements or even to address potentially inappropriate assumptions but rather to provide first lines of approach one could draw inspiration from.

## 6. Model Risk aggregated at risk type level

In terms of model risk measurement, one key question is regarding the level of the measure. Should a bank compute a measure at model level, at risk type level or at global level? In terms of feasibility, we believe that the most appropriate level on which to apply a methodology of model risk measurement is at risk type level. Indeed, a Model Risk Management framework needs to be able to measure its risks and to steer them in accordance with its model risk appetite, suggesting that a measure at global level would be appropriate. However, there are significant discrepancies between risk types particularly when it comes to encompassing non-risk models as well. Therefore, a model risk measurement methodology applied directly at global level would not be appropriate. On the other hand, applying a model risk measurement methodology at model level could be appropriate but would be highly time-consuming. Moreover, it would create an overlap with the Model Risk Assessments and the reviews performed by validation teams, both at model level. From our point of view, those different works are complementary and should leverage on one another, but their objectives remain different. While Validation work can be seen as the full verification that models fulfil the prerequisites to be used, Model Risk Assessment is a second step following validation whose goal is more to analyse, rate and rank models. Model Risk Assessment objective is rather to enable high level analyses in order to identify areas of improvement applying to several models for instance or even a type of model for which performance is considered low or unstable; which would need more monitoring for example. Since Model Risk Assessment is performed at model level, it should leverage on model reviews performed by validation teams. Of course, this description is only schematic and does not encompass every aspect of the work performed by validation teams or every use of Model Risk Assessments. Lastly, the measurement of model risk is one level up and one of its main purposes is to steer the risk at global level in accordance to the model risk appetite framework of the institution.

### 6.1. Indicators at risk type level

One can consider that a financial institution faces a model risk if it encounters one of the model risk situations introduced in section 2 at risk type level (Credit, Counterparty, Market, Operational, ALM, etc.). In this section, we focus on the second model risk situation, when the realised loss is larger than the expected one (including the margins of conservatism), at aggregated level. Figure 10 below illustrates several possible cases, at model level and at aggregated risk level for a given risk type.

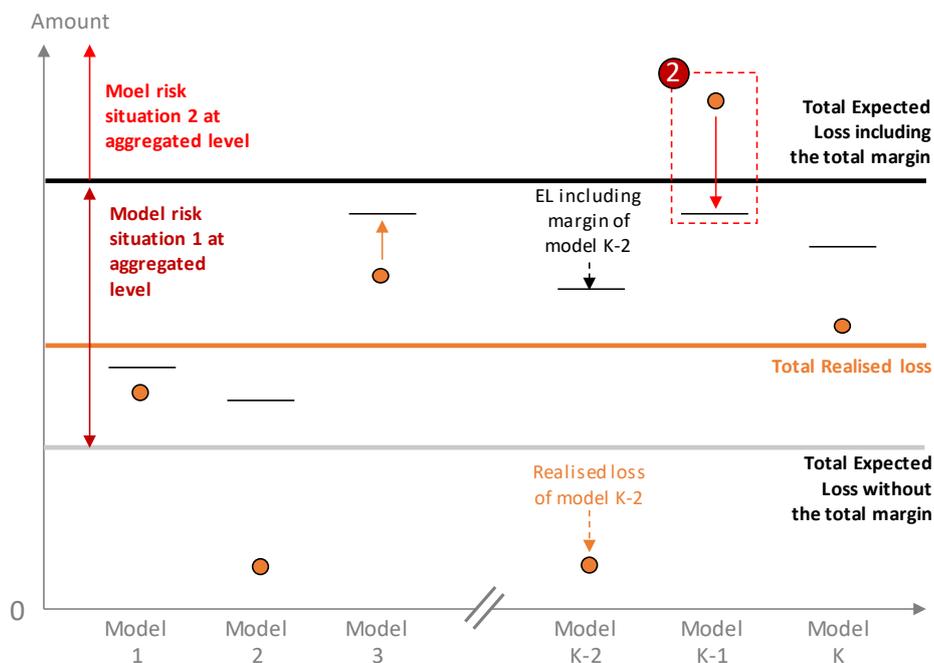


Figure 10 | Model Risk at individual model and at aggregate levels

Source: CH&Co GRA

One can notice that except for Model K-1, the gap between the expected loss and the realised ones of each model is positive, the expected loss (including the margin) being larger than the realised loss over the last period. Regarding model K-1, a model risk situation 2 is observed since the realised loss of this model is larger than the expected loss margin included. In order to analyse the model risk at aggregate level, considering K models as in the illustrative example above, the aggregated expected loss, the aggregated realised loss, the aggregated prudence margin and the aggregated gaps are valued as follows:

- **Total Expected Loss:** corresponds to the sum of the expected losses, including the prudence margins, of each model.

$$EL_{Total}^{Margin} = \sum_{k=1}^K Expected\ Loss_k + \sum_{k=1}^K \hat{M}_k^{Amount}$$

- **Total Realised Loss:** corresponds to the sum of the realised losses corresponding of each expected loss estimation.

$$Realised\ Loss_{Total} = \sum_{k=1}^K Realised\ Loss_k$$

- **Total Prudence Margin:** corresponds to the sum of the prudence margin of each model.

$$\hat{M}_{Total}^{Amount} = \sum_{k=1}^K \hat{M}_k^{Amount}$$

And

$$\hat{M}_{Total} = \frac{1}{EL_{Total}^{Margin}} * \sum_{k=1}^K \hat{M}_k^{Amount}$$

- **Total Gap:** corresponds to the difference of the total expected loss including margins and the total realised loss, divided by the total expected loss including margins:

$$D_{Total} = \frac{EL_{Total}^{Margin} - Realised\ Loss_{Total}}{EL_{Total}^{Margin}}$$

With  $D_{Total}$  assumed to be a random variable following a normal distribution (Central Limit Theorem)  $D_{Total} \sim \mathcal{N}(\hat{M}_{Total}, \sigma_{D_T})$ . The gaps observed on individual models belonging to the same risk type can thus offset one another at aggregated level. In the illustrative example displayed in Figure 10, since the aggregated realised loss is larger than the aggregated expected loss without the total margin of conservatism and below the one including the total margin of conservatism, it corresponds to a model risk situation 1 at risk type level.

## 6.2. Computation of a model risk measure at aggregated level

In the previous sections, the focus was made on the probability of occurrence. However, to be able to provide a “value” of model risk, the measure needs to be supplemented by other parameters, similarly to the computation of a Credit risk Expected Loss estimator depending on PD parameter but also combined with LGD and EAD parameters. Therefore, in the same vein as for Credit risk provisions, an appropriate measure of model risk could be the expected loss associated to a model risk situation. Hence, in order to measure the risk associated to the model risk situation 2 «  $RM(2)$  », one can estimate its expected loss «  $EL_{RM(2)}$  » as below:

$$\begin{aligned} EL_{RM(2)} &= \mathbb{E}(Realised\ Loss_{RM(2)}) \\ &= \mathbb{E}(Exposure_{Amount} * Loss\ rate_{RM(2)} * \mathbb{1}_{\{RM(2)\}}) \end{aligned}$$

Assuming those three components are independent and relying on the aggregated gap variable  $D_{Total}$ , the formula can be rewritten as follows:

$$\begin{aligned} EL_{RM(2)} &= \mathbb{E}(Exposure_{Amount}) * \mathbb{E}(Loss\ rate_{RM(2)}) * \mathbb{E}(\mathbb{1}_{\{RM(2)\}}) \\ &= MRE * \left| \mathbb{E}(D_{Total} | D_{Total} \leq 0) \right| * \mathbb{P}(\{D_{Total} \leq 0\}) \\ &= MRE * LGModRisk_2^{Agg} * ModRisk_2^{Agg} \end{aligned}$$

Where  $MRE = Model Risk Exposure$  and depends on the risk type measured. Thus, the MRE of the credit risk could correspond here to the sum of the Exposure At Default (EAD) of all their models.

$LGModRisk_2^{Agg}$  corresponds to the loss parameter in percentage following the occurrence of a type 2 model risk (“Loss Given Model Risk type 2”). And  $ModRisk_2^{Agg}$  still corresponds to the probability of occurrence of a type 2 model risk, but at aggregated level (risk type).

As in section 2 for individual models, by relying on  $D_{Total} \sim \mathcal{N}(\hat{M}_{Total}, \sigma_{D_T})$ , one can obtain the probability of occurrence of the model risk situation 2 at aggregated level:

$$\begin{aligned} ModRisk_2^{Agg} &= \mathbb{E}(\mathbb{1}_{\{D_{Total} \leq 0\}}) \\ &= \mathbb{P}(\{D_{Total} \leq 0\}) \\ &= \mathbb{P}(\{\hat{M}_{Total} + \sigma_{D_T} * \xi \leq 0\}) \\ &= 1 - \mathbf{N}\left(\frac{\hat{M}_{Total}}{\sigma_{D_T}}\right) \end{aligned}$$

By relying on this same distribution, one can also compute the  $LGModRisk_2^{Agg}$  parameter, as below:

$$\begin{aligned} LGModRisk_2^{Agg} &= \left| \mathbb{E}(D_{Total} | D_{Total} \leq 0) \right| \\ &= \left| \hat{M}_{Total} - \frac{1}{\mathbf{N}\left(\frac{-\hat{M}_{Total}}{\sigma_{D_T}}\right)} * \frac{\sigma_{D_T}}{\sqrt{2\pi}} * e^{-\frac{1}{2}\left(\frac{-\hat{M}_{Total}}{\sigma_{D_T}}\right)^2} \right| \end{aligned}$$

Indeed, with  $X \sim \mathcal{N}(\mu, \sigma^2)$  one can obtain the following formula:

$$\begin{aligned} \mathbb{E}(X | X \leq 0) &= \int_{-\infty}^{+\infty} x f_{X|X \leq 0}(x) dx \\ &= \mu - \frac{1}{\mathbf{N}\left(\frac{-\mu}{\sigma}\right)} * \frac{\sigma}{\sqrt{2\pi}} * e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma}\right)^2} \end{aligned}$$

Therefore, by combining the three parameters, one can obtain the expected loss related to type 2 model risk. For instance, considering the credit risk perimeter and using the Exposure At Default as the model risk exposure metric, one would obtain the following result:

$$\begin{aligned} EL_{Credit Risk}^{RM(2)} &= MRE * LGModRisk_2^{Agg} * ModRisk_2^{Agg} \\ &= EAD_{Total} * \left| \mathbb{E}(D_{Total} | D_{Total} \leq 0) \right| * \mathbb{P}(\{D_{Total} \leq 0\}) \\ &= EAD_{Total} * \left| \hat{M}_{Total} - \frac{\sigma_{D_T} e^{-\frac{1}{2}\left(\frac{-\hat{M}_{Total}}{\sigma_{D_T}}\right)^2}}{\sqrt{2\pi} \mathbf{N}\left(\frac{-\hat{M}_{Total}}{\sigma_{D_T}}\right)} \right| * \left( 1 - \mathbf{N}\left(\frac{\hat{M}_{Total}}{\sigma_{D_T}}\right) \right) \end{aligned}$$

In the same vein, one could also compute an expected loss based on the formulas that considers external effects impacting the distribution parameters or depending on the last gap value, or another method, combining the two of them for instance.

## Conclusion

Depending on the type of activity and/or the portfolio considered, one could use one method or the other, or even a combination of them. Regarding the choice of method to apply and the calibration of the different parameters of the distribution (for which enough historical data would hardly be available), they both should be expert-based in order to reflect the builders and users' views on the models and their risks. For instance, the calibration of the distribution law parameters should reflect potential changes over time in the portfolio structure. One must keep in mind that a model risk management framework has, as one of its main objectives, to involve the builders and the users of models in their risk management, as well as to ensure that the risk arising from the use of their models is well known, understood, accepted and managed by all the people involved.

In terms of amounts, one can bear in mind that the probability of occurrence of a model risk, within the meaning of this paper's type 2 model risk, as well as the expected loss related to type 2 model risk, are both expected to be very low at an aggregated level. In fact, the more models, the closer to 0 those metrics are expected to be, especially if the associated Exposure At Default are evenly distributed throughout the models.

Finally, we recall that one of the intentions of this paper is to provide "a model risk value" to compare to what banks currently allocate for this risk. Hence, one could use one of the methods outlined within this paper, calibrate the parameters with the appropriate experts and analyse whether the current model risk funds seem appropriate to cover this risk for the upcoming year(s).

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